MINISTRY OF EDUCATION
TE TĀHUHU O TE MĀTAURANGA

4 February 2022

[redacted]

Tēnā koe [redacted]

Thank you for your email of 20 November 2021 to the Ministry of Education requesting the following information in relation to the Briefing Note: Supplementary expert advice to inform the refresh of the mathematics and statistics learning area of The New Zealand Curriculum (1272703):

A. *The full version of the final supplementary report received on 17 September 2021*
B. *All annexes (1, 2, 3 and 4) from the brief to Minister Tinetti I referenced above that weren't proactively released*
C. *Any advice, briefs or memorandums prepared for the Education Secretary or anyone in the Ministry of Education senior leadership team about the Royal Society Te Aparangi expert panel report on mathematics/statistics, the rationale for a supplementary report and advice, briefs or memorandums on the supplementary report.*
D. *Any critiques, commentary or discussion, regardless of its form, Ministry of Education staff produced regarding the Royal Society Te Aparangi expert panel report on mathematics/statistics and the supplementary report received on 17 September 2021.*

Your request has been considered under the Official Information Act 1982 (the Act). Our response to each part of your request is outlined below:

Part A: The full version of the final supplementary report received on 17 September 2021

We are releasing this document in part, with some information withheld under section 9(2)(a) of the Act, to protect the privacy of natural persons, and section 9(2)(ba)(i) of the Act, to protect information shared under an obligation of confidence. This is attached as **Appendix A**.

Part B: All annexes (1, 2, 3 and 4) from the brief to Minister Tinetti I referenced above that weren't proactively released

Annex 1: *Report to the Ministry of Education: Mathematics and statistics skills and knowledge learners need to know by when, important cross-disciplinary links, and considerations in light of rapid changes and growth in computer science/ICT. Authors: Fiona Ell (Auckland University), Robin Averill (Victoria University) & Jane McChesney (Canterbury University)*

This is the same report as requested in Part A of your request, attached as **Appendix A**.

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Annex 2: *Royal Society Te Apārangi media release statement in relation to the RSTA independent paper in preparation for public release on 3 October 2021.*

This part of your request has been refused under section 18(d) of the Act, as the requested document is publicly available on the Royal Society Te Apārangi's website at the following link: https://www.royalsociety.org.nz/news/independent-report-on-improving-maths-and-stats-learning-finds-investment-and-changes-needed-at-virtually-all-levels-of-the-education-system/.

Annex 3: *Reactive Q&A sheet for Ministry use which have been provided to you previously [METIS 1271858 refers]*

We are releasing this document in full, attached as **Appendix B**.

Annex 4: *Q&As for Minister use*

We are releasing this document in full, attached **Appendix C**.

Part C: Any advice, briefs or memorandums prepared for the Education Secretary or anyone in the Ministry of Education senior leadership team about the Royal Society Te Aparangi expert panel report on mathematics/statistics, the rationale for a supplementary report and advice, briefs or memorandums on the supplementary report

No advice, briefs or memoranda have been prepared in scope of this part of the request. As such, this part of your request has been refused under section 18(e) of the Act, as the documents alleged to contain the information requested do not exist.

Part D: Any critiques, commentary or discussion, regardless of its form, Ministry of Education staff produced regarding the Royal Society Te Aparangi expert panel report on mathematics/statistics and the supplementary report received on 17 September 2021

We have identified one document within scope of your request. We are releasing this document in part, attached as **Appendix D**, with some information withheld under section 9(2)(g)(i) of the Act, to maintain the effective conduct of public affairs through the free and frank expression of opinions by or between officers and employees of any public service agency or organisation in the course of their duty.

Please note, the Ministry now proactively publishes OIA responses on our website. As such, we may publish this response on our website after five working days. Your name and contact details will be removed.

You have the right to ask an Ombudsman to review this decision. You can do this by writing to info@ombudsman.parliament.nz or Office of the Ombudsman, PO Box 10152, Wellington 6143.

Pdleave

Associate Deputy Secretary
Te Mahau | Te Poutāhū (Curriculum Centre)

Report to the Ministry of Education

Mathematics and statistics skills and knowledge learners need to know by when, important cross-disciplinary links, and considerations in light of rapid changes and growth in computer science/ICT

September 2021

Prepared by Robin Averill, Fiona Ell, and Jane McChesney

Contents

Report focus areas

The focus areas requested for this report were:

- The mathematics and statistics skills and knowledge learners need to know by when
- Important cross-disciplinary links between mathematics, statistics and other discipline areas
- Considerations regarding rapid changes and growth in computer science/ICT

Overview of the report

In the report we describe the ways we worked, the national context, recommendations in light of the signalled bicultural nature of the curriculum refresh, rationale for the organisation of mathematics content and key recommendations for organisation of mathematics and statistics content across the year levels, explanation of the template used to show year-by-year mathematics and statistics focus points, discussion of important cross-discipline links for mathematics and statistics, considerations regarding rapid changes and growth in computer science, consideration regarding other aspects of pedagogy, and final comments. Year-by-year indications of mathematics and statistics focus points, along with suggested overview purposes of each part of the mathematics and statistics curriculum are included.

Process

The task was explained to the team in a one-day hui at the Ministry of Education on 13 August 2021, followed by the team planning an approach for working together and sharing tasks. Collaboration continued over the subsequent two weeks with frequent email and Zoom discussions to clarify process matters and agreed responses to the tasks set. Our main reference points for developing the report have been the current and previous iterations of the New Zealand mathematics and statistics curriculum, international examples, literature, and our varied experiences locally, nationally and internationally within mathematics and statistics education. Consultation possibilities were constrained by the timeframe given for the preparation of the report and the country being in level 4 lockdown over this time. However, we are very grateful to Dr Karyn Saunders, Dr 9(2)(a), Dr Pip Arnold and Anna Fergusson for their very kind and swift feedback on selected ideas within the year-by-year suggested content.

National context of mathematics and statistics education

The report was completed within a time of substantial change and consultation across a wide range of general and mathematics and statistics-specific national education developments, such as the NCEA review, development and piloting of numeracy NCEA co-requisites, the curriculum refresh, and a Royal Society expert panel report on mathematics and statistics education. The report team have had varying involvements in these developments but carried out the work for this report independent of but with some consideration towards these other initiatives and developments. Furthermore, *Tātaiako* (Education Council, 2011) and *Tapasā* (Ministry of Education, 2018) have been published and advocated for teacher use since the 2007 national curriculum was written, and the mathematics and statistics curriculum has not yet been revisited in light of these significant documents and related research, professional development and understandings in the area of culturally sustaining practice.

Mathematics and statistics in a bicultural curriculum

The curriculum refresh is intending a bicultural curriculum[1]. This report was completed without guidance on what it is intended for a bicultural mathematics and statistics curriculum. The starting points for our work are Euro-centric rather than bicultural by design. Since the 2007 New Zealand curriculum came into effect, kaiako have been expected to respond in their teaching to *Tātaiako* and a new professional code and standards (Education Council, 2017), both of which enhance the place of te reo me te ao Māori in their work, but are not explicit about what this means for the teaching of mathematics and statistics. We have tried to include some emphasis consistent with a bicultural curriculum. However, we are writing from the position of Tangata Tiriti, not Tangata Whenua. Tangata Whenua voices are clearly essential to determine what mathematics and statistics are needed by when, cross-curriculum links, and the impacts of rapid ICT growth, for a bicultural mathematics and statistics curriculum. The basic concept of what 'progress' might be is itself a cultural construct. For the curriculum to be bicultural it should have bicultural origins and originating frameworks which we cannot provide. Control over what is in the mathematics and statistics curriculum should be fundamentally shared. Our contribution is made to help progress the Ministry of Education's work, but it needs to be considered in light of this fact.

Recommendations:

- We strongly advocate for further work to discuss and describe what is meant by a bicultural mathematics and statistics curriculum, including what mātauranga Māori and mātaraunga-a-iwi is included when and how it is included, before writing of the refreshed mathematics and statistics curriculum begins.

[1] https://www.education.govt.nz/our-work/changes-in-education/national-curriculum-refresh/new-zealand-curriculum/

We recommend further work occurs to:

- reconsider what mathematics and statistics are needed by ākonga by when in light of the recommendation above
- consider further ways in which the curriculum will be written, portrayed and implemented as a bicultural curriculum (e.g., considering protocols, pedagogies, cultural competencies, te reo Māori, learning environment contexts, contexts of learning activities, awareness of how people have been involved in the development and use of mathematical and statistical ideas and tools over time, involvement of whānau and families in supporting learning, developing and nurturing culturally-linked values)
- discuss, decide, and describe ways to support kaiako in designing bicultural mathematics and statistics learning opportunities for ākonga
- consider consultation and communication with whānau, families, and society needed for supporting curriculum implementation.

Other considerations regarding culturally sustaining mathematics and statistics

Alongside consideration of mātauranga Māori and mātauranga-a-iwi within mathematics and statistics content, it is vital to consider responsibilities to our learners with Pacific heritage in relation to curriculum content and emphasis. Links between *Tātaiako* and *Tapasā* and mathematics and statistics must be made explicit by curriculum writers. We note and are supportive of the framing of mathematics and statistics and links to the important ideas of these documents in the draft NCEA numeracy corequisite materials[2], as in the 'Numeracy Learning Matrix' and the 'Unpacking Numeracy' document (p. 1) and in the table showing links between mathematical process ideas, the key competencies, *Tātaikao* and *Tapasā* (final page of Unpacking document). The table makes links between mathematical processes and these competencies.

Recommendation:

- We strongly advocate consideration of mathematics and statistics content in light of *Tātaiako* and *Tapasā* and related knowledge and understanding about the groups

[2] https://ncea.education.govt.nz/numeracy-0 and
Australian Curriculum, Assessment and Reporting Authority. (2021). *Mathematics consultation curriculum, All elements F-10.*
https://ncea-live-3-storagestack-53q-assetstorages3bucket-2o21xte0r81u.s3.amazonaws.com/s3fs-public/2021-08/Numeracy%20Learning%20Matrix%20%28A4%29.pdf?VersionId=Nw0dtOQPEF6bNm8Z6YooJBYasvB47udf
https://ncea-live-3-storagestack-53q-assetstorages3bucket-2o21xte0r81u.s3.amazonaws.com/s3fs-public/2021-08/Unpacking%20Numeracy%20%28A4%3B%20Page%206%20is%20A3%29.pdf?VersionId=j.cT_3zjOJWnrwHFdOe83PbIKltnDj4b

these documents are seeking to serve. Further, these links need to be made explicit and visible in curriculum materials to help teachers see and use them.

The mathematics skills and knowledge learners need to know by when: Years 0-13

We need to be specific about the mathematics and statistics that we want ākonga to learn each year they are at school because wider bands (e.g., such as the current curriculum levels, or considering group of years such Years 0-3, 4-6 etc) make it hard for teachers to know what they should be aiming for and whether ākonga have made appropriate progress for further success. Making local curriculum is a complex task for teachers and clear guidelines will assist.

Year-by-year structure

Curriculum documents serve a number of purposes in education systems. By their structure, as well as their content, they convey important messages about what is valued and how knowledge, skills and attitudes are perceived. Some purposes for curriculum are well-served by high level, broad and open documents, which provide space for local interpretation. Other purposes are well-served by providing more specific information that provides clear direction. Both approaches can be used together within a curriculum to serve both sets of purposes.

In mathematics and statistics education in English medium schools in Aotearoa we have three key issues made evident considering international comparative studies:

- There is dissatisfaction with the achievement profile and rate of progress of ākonga. Ākonga are not learning enough mathematics and statistics, and they are not learning it soon enough.
- There are major equity problems in mathematics and statistics achievement and progress, and engagement and participation. There are larger gaps between those who do well and those who do not than in other countries, Māori and Pacific learners are not getting sufficient access to learning experiences that enable them to achieve in the same way as Pākehā and Asian ākonga, and SES has a bigger impact on achievement and progress than in other jurisdictions, with schooling making less of an impact on the effects of poverty than in other, comparable, places[3].
- Variability in ākonga outcomes on PISA tests is attributed to 'within school' factors more than 'between school' factors in Aotearoa. Our 'within school' variability is amongst the highest in the PISA jurisdictions[4]. NMSSA data also shows that ākonga

[3] Ministry of Education. (2020). https://www.educationcounts.govt.nz/__data/assets/pdf_file/0015/205710/TIMSS-2018-Maths-Achievement-A4.pdf
[4] OECD. (2019). https://www.oecd.org/pisa/PISA%202018%20Insights%20and%20Interpretations%20FINAL%20PDF.pdf

outcomes in mathematics and statistics are shaped by school decile[5]. Reducing within-school and between-school variability is a key to improving ākonga outcomes.

With the opportunity to refresh the mathematics and statistics curriculum we need to ask: what sort of curriculum structure will best address these key issues? While the curriculum structure is not the most immediate influence on teaching and learning, it provides a crucial framework for how teachers, ākonga and communities think about mathematics and statistics teaching and learning[6]. The structure needs to be the one most likely to result in:

- Sufficient progress and achievement to meet our goals
- Equity in access and in outcomes
- Reduction in variability of learning experiences and achievement opportunities

There is very little (no) direct research on how curriculum structure impacts system outcomes. There is some work on how teachers work with materials to design learning, but there is not a large body of evidence about how this process occurs between national curriculum statements and classroom programme design and implementation[7]. What evidence there is suggests that teachers can learn about both mathematics and statistics content and how it should be staged or approached through interacting with curriculum documents[8].

To decide what curriculum structure might best promote achievement, progress, equity and consistency across the system we therefore have to turn to thinking about what it is that teachers and leaders need in order to provide opportunities for learning mathematics and statistics that will achieve these aims. Teacher knowledge of mathematics for teaching is recognised as a significant factor in quality of mathematics teaching[9]. One way to support teacher knowledge of mathematics for teaching is to provide a curriculum that clarifies for teachers what they should be teaching. In presenting clear curriculum direction and specificity, teachers have access to

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[5] NMSSA. (2018). https://nmssa.otago.ac.nz/reports/2018/2018_NMSSA_MATHEMATICS.pdf

[6] Schmidt, W. H., & Prawat, R. S. (2006). Curriculum coherence and national control of education: Issue or non-issue? *Journal of Curriculum Studies, 38*(6), 641–658.

[7] Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.
Sullivan, P., Clarke, D., Clarke, D., Farrell, L., & Gerrard, J. (2013). Processes and principles in planning mathematics teaching. *Mathematics Education Research Journal, 24*(4), 457-480.

[8] Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*, 331-353.
Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.

[9] Ball,D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59* (5), 389-407.
Charalambous, C. Hill, H., Chin, M., & McGinn,D. (2020). Mathematical content knowledge and knowledge for teaching: exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education, 23*, 579-613.
Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12-24.
Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*, 255-281.

information about mathematical content at a level of detail that expands/strengthens their knowledge bases for learning[10]. In particular, a curriculum can draw together evidence from developmental psychology, neuroscience, mathematics education, statistics education, mathematics and statistics disciplines, and other national priority areas to form outcomes for learners that highlight:

- Sequences for learning
- What is important for progress and future learning
- Connections among different ideas and processes

A curriculum should be a tool for teachers. It should help them be better at preparing for and teaching mathematics and statistics. The tool that a curriculum is most like is a map. The curriculum represents the 'terrain' of mathematics and statistics: the more detail that is included, the less likely you are to get lost. A map points out the important features of the landscape and shows you how to get there, and it shows how the different parts of the landscape relate to each other. Research on learning trajectories provides a strong base for mapping learning pathways in mathematics and statistics[11]. Some of this work suggests that setting out curriculum expectations for each year provides a number of advantages over broader bands of year levels. One advantage is in terms of clearly identifying learning sequence, particularly when there is an element of learning needed for later topics (requisite knowing). For some elements of content, learning at later times (either within a year or in following years) provides opportunities to revisit and consolidate content (reaching into prior learning), and can extend or connect these important elements[12]. These features of year-by-year curriculum illustrate the benefits of mapping for coherence across and within mathematical content topics. When the curriculum does not provide enough information for teachers to find their way, they turn to other 'maps' to help them: in our case, examples include the Numeracy Project framework, the National Standards, the Learning Progression Framework, commercial textbooks, online mathematics resource collections (e.g., Achievement Objective 'elaborations') and online programmes[13]. It is timely to return to more balance where the level of detail in the curriculum provides a greater shared common curriculum space for all schools and teachers (a curriculum mathematics and statistics 'commons'), while still enabling scope for adapting and designing for localised school contexts[14].

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[10] Cobb, P., & Jackson, K. (2011). Assessing the quality of the Common Core State Standards for mathematics. *Educational Researcher, 40*(4), 183–185.

[11] e.g., Confrey, J., Maloney, A. P., & Corley, A. K. (2014). Learning trajectories: A framework for connecting standards with curriculum. *ZDM—The International Journal on Mathematics Education, 46*(5), 719–733. and Nguyen, K. H., & Confrey, J. (2014). Exploring the relationship between learning trajectories and curriculum. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. 161-185). Information Age Publishing.

[12] Rowland, T. (2014). Mathematics teacher knowledge. In P. Andrews & T. Rowland (Eds.), *Masterclass in Mathematics Education: International perspectives on teaching and learning* (pp. 87-98). Bloomsbury Academic.

[13] McChesney, J. (2017). Searching the New Zealand curriculum landscape for clarity and coherence: Some tensions in Mathematics and Statistics. *Curriculum Matters, 13*, 115-131.

[14] Prawat, R. S., & Schmidt, W. H. (2006), Curriculum coherence: Does the logic underlying the organisation of subject matter matter? In S. J. Howie & T. Plomp (Eds.), *Contexts of learning mathematics and science: Lessons learned from TIMSS* (pp. 265-276). Routledge.

In short, the 2007 NZC Mathematics and Statistics document does not have enough detail for teachers to plan Mathematics and Statistics programmes, and so they turn to other sources that give them enough detail, some created specifically for this purpose. This reaching outside the curriculum can be seen as a constructive process of teachers and schools taking local ownership of curriculum and meeting learner needs, but in practice teachers' knowledge of mathematics and statistics and the pressure experienced in teaching the full breadth of the curriculum means that it is more likely that they will adopt a proxy curriculum wholesale than they will craft a local programme. Mathematics and Statistics are not easy to teach, and there is a huge volume of information about how ākonga learn mathematics and statistics that teachers can never hope to master in the face of their other tasks. A more specific curriculum than the current 2007 curriculum is an opportunity to help teachers design and teach mathematics and statistics learning experiences that make a difference for all learners. Access to specific curriculum information, and opportunities to work together with colleagues, act as feedback loops for teachers who in turn consolidate their curricular and content knowledge for teaching[15].

Evidence from four contributing bodies of research can be used to make the decision to have a year-by-year curriculum rather than broader bands:

1. Early mathematics learning in primary school is very significant to future progress and has many small but important steps that could be missed if year-by-year detail is not provided.

There is a strong body of evidence that what ākonga learn early in their primary school mathematics journey is highly important for and predictive of their rate of progress and their achievement[16]. Particular concepts and skills have been highlighted that can be specified in a curriculum document and become part of everyday mathematics programmes[17]. The detail we have from research into learning trajectories[18] can be used to help teachers teach what is important and what will help all learners make good progress if it is included in the curriculum. There is also evidence that concepts in number and spatial relationships are 'tied together',

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[15] Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*, 331-353.

[16] Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21* (2), 33-49.

Rittle-Johnson, B., Fyfe, E., Hofer, K., & Farran, D. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development, 88* (5), 1727-1742.

Wilkins, J., Woodward, D., & Norton, A. (2021). Children's number sequences as predictors of later mathematical development. *Mathematics Education Research Journal, 33,* 513-540.

[17] Aubrey, C., Dahl, S., & Godfrey, R. (2006). Early mathematics development and later achievement: Further evidence. *Mathematics Education Research Journal, 18*(1), 27 – 46

Clements, D., Sarama, J., Spitler, M., Lange, A. & Wolfe, C. (2011). Mathematics learned by young children in an intervention based on learning trajectories: A large-scale cluster randomized trial. *Journal for Research in Mathematics Education, 42* (2), 127-166.

Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21* (2), 33-49.

[18] Clements, D., Sarama, J., Baroody, A. & Joswick, C. (2020). Efficacy of a learning trajectory approach compared to teach-to-target approach for addition and subtraction. *ZDM, 52*, 637-648.

particularly in early mathematics learning[19], and a more specific curriculum can trace these connections, making them clear, and make sure their potential for improving learning is maximised.

2. Teacher expectations are critical to ākonga progress, and without clarity about what is possible, teachers' expectations can be too low.

There is evidence that ākonga are capable of more than most teachers expect of them. There is also evidence that teachers tend to expect less of Māori and Pacific ākonga than they are capable of, and tend to expect less of them than they might of ākonga of other ethnicities[20]. This is significant because of the impact of teacher expectation on learner progress and achievement, especially in mathematics[21]. By providing specification of what is possible to learn we can support teachers to have high expectations of all learners. To accelerate progress, the curriculum should make clear that more is possible in the first two years of schooling. Hiding these enhanced expectations in a three-year band risks continuing the underestimation of our ākonga, leading to insufficient progress and achievement – and ongoing inequity.

Support for teacher planning and teaching relates to the grain-size of curriculum detail. The current levels structure of the NZC has been problematic because of the large-grained curriculum information in the Achievement Objectives. The additional information provided by the Elaborations describes and explains detail of content, rather than expressing a learning pathway through the level. A year-by-year curriculum plan however provides finer grained detail so that teachers can more easily identify prior learning from previous years and know that ākonga have been taught this content. At present with the NZC levels, teachers cannot be sure what ākonga have been taught in the previous year, and so spend a great deal of time and effort in assessing and re-teaching. Another advantage of more fine-grained curriculum detail identifies the sequence of progress in each strand as well as between the strands[22]. Greater connections between the strands are important and can be made explicit, providing a platform for deeper conceptual understanding.

3. Teacher knowledge of mathematics and statistics for teaching is uneven, and because that knowledge is fundamental to improving learner outcomes, the curriculum needs to support and build teacher knowledge.

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[19] Lowrie, T., Resnick, I., Harris, D., & Logan, T. (2020). In search of the mechanisms that enable transfer from spatial reasoning to mathematics understanding. *Mathematics Education Research Journal, 32,* 175-188.
Young-Loveridge, J. (2011). Rethinking the role of counting in mathematics learning. *Teachers and Curriculum, 12*, 79-83.
[20] Rubie-Davies, C. (2015). *Becoming a high-expectation teacher: Raising the bar.* Routledge.
[21] Rubie-Davies, C. (2015). *Becoming a high-expectation teacher: Raising the bar.* Routledge.
[22] Rowland, T. (2014). Mathematics teacher knowledge. In P. Andrews & T. Rowland (Eds.), *Masterclass in Mathematics Education: International perspectives on teaching and learning* (pp. 87-98). Bloomsbury Academic.

A more detailed and specific curriculum can support teachers' pedagogical content knowledge and knowledge of mathematics and statistics for teaching by organising and describing the key ideas in ways that support good design of learning experiences and formative assessment[23]. Quality opportunities to learn and feedback on learning are long-established tenets of equitable teacher practice, both of which require in-depth knowledge of content[24]. Providing a detailed framework for mathematics and statistics teaching can help teachers and leaders structure mathematics and statistics programmes in helpful ways. The clearer the curriculum is, the better targeted professional learning to improve teacher knowledge can be too.

4. The nature of mathematics and statistics as disciplines demands more detail than other curriculum areas

The structure of content (including process) for school mathematics and statistics is different to the structure of content (including process) for literacy, for example. Content of school mathematics and statistics has a structure that is both sequenced and interconnected. We draw an illustration from the number system (the parallel to the code of written language for literacy learning). Learning the code of how numbers are written is one part of the process, and the meaning of the number has to be understood in terms of its relative magnitude in the system of numbers. 28 represents a quantity but it is also in relation to one more than 27, and two less that 30 (where 30 is a benchmark number in the place value decimal structure of the number system). In literacy a word is a combination of letters that has a different relationship depending on context and meaning/s. For one more contrast with literacy, knowing to read from left to right and how to identify the letters of the alphabet (known as constrained skills) are claimed to be learned in a relatively short time. There is a much more generative aspect to literacy with more unconstrained learning needed for comprehension, speaking and writing. In contrast, when learning about number, ākonga learn about different types of numbers, and therefore different ways to write/encode numbers and number relationships. Knowing the place value system is not enough – there are decimal, fraction, negative numbers that make up a fundamental 'code' of numbers. The repertoire of constrained content in mathematics and statistics is substantial and typically needs to be sequenced across the years of the primary school.

We describe the mathematics and statistics experiences and understandings within *Te Whāriki* (Ministry of Education, 2017) that we can hope many students have on school entry, how we have considered the task of giving year-by-year indications of mathematics and statistics knowledge and understandings for Years 0-13, and key areas of focus leading these decisions. We used a guiding principle of each year level having some content that was identifiably new for ākonga (as well as material building on previous learning), and that teachers could extend learning

[23] Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*, 331-353.
Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.
[24] Hattie, J. & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81-112.

opportunities for ākonga within the year rather than moving into the next year level for extension ideas.

Many of the 'what by when' suggestions we offer in the year-by-year outlines below can be conceived of as whether or not ākonga are making links between powerful ideas, rather than whether they have or can perform from siloed items of knowledge. We need to check that ākonga can see these links, rather than viewing such signposts as simply not being able to perform specific skills. Key links, fundamental to sound number sense and good number understanding and arithmetic involve 'composite units' – things are made up of mathematical entities that can be split up and recombined according to rules – which can be numbers, shapes, patterns, seeing things and working on things in groups, and relative magnitude (e.g., whole numbers, rational numbers). The number line is a powerful model that can be used to understand composite units that also links to measurement scales and continuous models of number. Understanding that number is continuous, yet sometimes treated as discrete, and being able to see patterns and use them to understand and predict are all more significant for success in mathematics and statistics than just holding constituent knowledge or skills. Understanding of particular mathematics and statistics processes – such as ways of thinking, solving 'problems', communicating mathematically and statistically – and key competencies and cultural competencies are also important to embed throughout, so that the use and usefulness of mathematics and statistics are paramount, rather than separate, seemingly disparate, skills and strategies.

Students being able to see and make connections between mathematics and statistics ideas across the mathematics and statistics curriculum is associated with mathematical confidence, competence, understanding and engagement. Our suggestions of what mathematics and statistics is needed by when and our explanations of these demonstrate the importance of ākonga seeing and feeling confident about the interconnectedness of mathematics and statistics ideas across strands of the curriculum.

Reporting of 'what by when' - Explanation of style and format

Signposts

At each year level we have identified both the broad mathematical and statistical landscape for that year level and key learning that is required/needed/crucial for continued learning in the following or later years. We have named the key learning points as 'signposts' - places in the learning landscape that need particular attention and consolidation for future successful learning. The role of these signposts is to guide teachers when teaching this aspect of the curriculum - to concentrate attention but not take away learning time from other topics. As described above, the signposts give important direction, but their links to each other and to surrounding parts of mathematics and statistics are also critical for ākonga success.

A template used to organise presentation of these ideas was developed collaboratively by the report team. Each year level's version of the template provides:

- overall emphasis for each strand each year
- a sketch of the mathematics and statistics landscape for that year, in the form of learning outcomes organised by strands (shown in normal text), preserving the sense of a rich learning experience
- ‘what by when’ aspects we feel would most hinder ākonga making further mathematics and statistics learning progress (signposts shown in bold)

Because we are concerned that the signposts could be seen as a checklist of skills we have placed them in their proper context in the template by:

- indicating the pervasive emphasis on mathematical processes, and
- showing a desire for a year-by-year thought/motto/whakataukī/kīwaha as a driving metaphor or statement of the year level’s key mathematical and statistical contributions to the ākonga’s learning journey.

Guiding focus for each year level

The 2007 curriculum was the first time that inclusion was made of a guiding whakataukī for our learning area, with one whakataukī to guide all levels. To build on this, a suitable next step would be to expand into adding a guiding whakataukī or motto, saying, kīwaha or the like for each year level. Each guiding whakataukī or motto, saying, kīwaha could be an inspirational focus for the mathematics and statistics teaching and learning at each year level that could be accompanied by a short paragraph highlighting key messages about mathematics and statistics teaching and learning of the year. These would ideally be determined as part of the curriculum writing process.

Preparing the templates

To start our work, each of the team completed the template for Year 7. Time has not been sufficient to develop shared views on all year-by-year content or to make all the year-by-year reporting similar in style. Rather, we offer the variety in style as each provides examples of what we have been asked to provide in different and useful ways. We completed templates for each year level, utlising the different knowledge and experience of the team. The templates are included below and overviews of continuity of overall ideas for each strand for each year level, taken from these, are included in Appendix One.

We have not carried out in depth consideration of the mathematical processes in the ‘what by when’ exercise or reporting, and hence these are left in each year-by-year overview as placeholders for the curriculum writers’ further consideration. We believe there should be associated consideration of ‘what-by-when’ related to aspects outside of mathematics and statistics content, such as in progress with using mathematical processes, metacognitive processes, in mathematical and statistical self-efficacy, in literacy required for interpreting and solving problems, and in relation to engagement, for example. This work will be essential for ensuring ākonga success in the refreshed curriculum.

To give some indications of ways to consider the mathematical processes, examples are presented as three ‘stems’ where each combines important and related processes, to describe that we act and think mathematically and statistically in these ways, by:

- Making sense of and solving problems and using models, with perseverance
- Reasoning and arguing mathematically and statistically, with precision, and critiquing others’ reasoning
- Looking for, making use of, and communicating about patterns, regularity, and structure

These examples for describing mathematical processes encapsulate most of the key competencies in the 2007 curriculum, and in particular, the two competencies most closely connected with learning area content; ‘thinking’ and ‘using language, symbols, and texts’. The key competency of ‘thinking’ encompasses ways of thinking mathematically including being creative, being critical and being curious, contributing to success and enjoyment of learning. Not only are these important dispositions or orientations towards learning, for ākonga this can position them as active thinkers, of contributors to knowledge building in class settings, and for their engagement, belonging and connectedness. This focus also connects with current ideas of ‘future focused’, life-long learning, and being an engaged and critical citizen. The key competency of ‘using language, symbols, and texts’ is illustrated when ākonga at all years use both informal and formal language and symbolic forms in their mathematical activity. In a mathematics and statistics context, ‘encoding or decoding a text’ (using a literacy-based definition) relates to writing and reading texts in the form of sequences, equations, graphs (both algebraic and statistical), diagrams and other modelling representations. All of the above are encapsulated in the three mathematical processes and are also evident in the content included in the year-by-year templates below. The mathematical processes are pivotal to ākonga learning and signposts inclusive of processes and competencies are needed because the processes drive learning, engagement, and achievement. We have not had sufficient time to give full attention to signposts for the processes. However, some examples from a range of the year-by-year templates that illustrate how the mathematical processes are profoundly connected to important learning include:

In Year 3 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance: Use array models, number lines and materials that model the place value system to support problem solving and build understanding

Reason and argue mathematically and statistically, with precision, and critique others’ reasoning: Explain actions and ideas, ask another person for their ideas and be able to summarise what they said in reply

Look for, make use of, and communicate about patterns, regularity, and structure: Talk about and use array structures, the patterns in the place value system and patterns in basic facts, families of facts and multiples

Examples of signposts at Year 3 (from Year 3 template)

Number

Multiply single digit numbers and model the process using array models and other spatial models

Compare any two amounts and say which is bigger and by how much

Geometry
Make, copy and describe patterns that involve symmetry and rotation

Statistics
Asking questions and making a plan, gathering data, sorting data and explaining their reasoning

In Year 6 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance: Especially problems that embed multiplicative relationships and situations
Reason and argue mathematically and statistically, with precision, and critique others' reasoning: Explain, listen, summarise, question, build and critique ideas
Look for, make use of, and communicate about patterns, regularity, and structure: especially patterns in how the operations work and things can be seen as 'the same' in different ways, depending on what properties you are attending to

Examples of signposts at Year 6 (from Year 6 template)

Algebra
Use a table they are given to organise an explanation of a pattern that relates the term of the pattern to the pattern sequence (functional rather than recursive thinking)
Geometry
Use a co-ordinate system or the language of direction and distance to specify locations and describe paths
Statistics
Looking for patterns in the data, identifying the spread of data, and identifying and discussing patterns and trends in the data, (spread and outliers) and communicating our findings in text and visually

Rationale for content organisation: Number/Measurement, Algebra, Geometry and Statistics and Probability

Number and Algebra are individual strands so that each has a focus that can be differentiated and strengthened. Geometry is one strand - again to focus in on geometry content and the learning and teaching of geometry. Measurement has been moved to be considered within the Number strand. The rationale for this shift is that much of measurement learning relates to 'number in action', that is, the quantifying of measurements. This quantifying involves such actions as counting, repeating units in ones or tens, hundreds etc., (following the number system), reading scales (forms of number lines), and comparing measures (relative aspects of the attribute). Measurement provides both multiple contexts for using number (e.g., decimal numbers and parts of 'units' or wholes), and a system of measurement units based on the base ten number system (e.g., millimetres, centimetres, metres). This organisation brings discrete and continuous ways of quantifying into one strand, while highlighting powerful connections with other strands. For example, the measurement attributes of length, area and volume connect with spatial concepts/knowledge and visualisation. Counting and measurement data are part of the Statistics and Probability strand. Angle is part of the Geometry strand. Learning to 'tell the time' involves reading and interpreting numbers, and, for analogue clocks, connects with language and visualisation of simple fractions. The measurement processes of using, reading and making sense of scales, as well as knowing how to use measurement instruments are elements of mathematical practices of using tools and representations. Measurement processes also illustrate cultural experiences of engaging with and coming to know a sense of place – for example, a step or pace is known as hikoi, a concept also meaning shared walking together from one place to another for an important purpose. Similarly, aspects of 'measurement' sense such as using benchmarks (referents) to estimate measurements are mostly related to repeated whole or parts of the benchmark - whether by prior experience of the measurement of the benchmark (e.g., handspan) or using visual information that is at hand.

Rationale for inclusions and use of bold to show key signposts

We see what we are proposing as 'what by when' signposts in the templates as minimums rather than as targets. We want teachers to be encouraged and free to do more within the year, and not to do less than what has been suggested. We recognise that each part of the signposting has beginnings somewhere before the level it is included, which may or may not be explicit in the suggestions, and in most cases, later understanding will grow from aspects included earlier. One way we have framed the focus areas of most concern for further progressions is to show these in bold. A difficulty here is that a list of this sort might imply a narrowed curriculum, a check list of skills or high-stakes assessment foci, rather than being seen as signposts in a broad and rich mathematical and statistical landscape. Without a landscape to traverse, signposts are purposeless. In the same way, signposts important for progression are purposeless if they are not situated within a rich mathematical and statistical learning experience that creates interesting and valued pathways between the signs.

(i) Early Childhood Mathematics and Statistics – Prior to Year 0/1[25]

Years 0-13 mathematics builds on mathematics and statistics experiences and learning children have prior to entering school. Mathematics is an identifiable part of *Te Whāriki* the English version of our national early childhood curriculum. In contrast to the *The New Zealand Curriculum* (Ministry of Education, 2007) that groups understandings about the world in learning areas, in *Te Whāriki*, these understandings are woven through the strands of Mana Atua – wellbeing, Mana Whenua – belonging, Mana Tangata – contribution, Mana Reo - communication, and Mana Aotūroa – exploration. Mathematics is "explicit in communication and exploration" and "implicit in other strands" (p. 52). 'Young children' are the ages of approximately two and a half years to school entry, and their "developing literacy and mathematical abilities embrace new purposes, such as reasoning, verbal exploration, puzzling and find out about the physical and social world" (p. 15).

Examples of mathematically oriented Learning Outcomes in *Te Whāriki* include:

> Recognising mathematical symbols and concepts and using them with enjoyment meaning and purpose | he kōrero pāngarau (p. 42)
>
> Playing, imagining, inventing and experimenting | te whakaaro me te tūhurahura I te pūtaiao (p. 47).

[25] This material is based on the current English version of *Te Whāriki* (Ministry of Education, 2017) and other early childhood mathematical resources from the Ministry of Education, such as *Te Kākano* (Kei Tua o te Pae, Book 18, *Mathematics Pāngarau,* Ministry of Education, 2009) and *Te Aho Tukutuku* (2010).

Using a range of strategies for reasoning and problem solving | te Hīraurau hopanga (p. 47)

Examples of evidence of learning and development related to mathematics communication and exploration for Young children include:

Familiarity with numbers and their uses by exploring and observing the use of numbers in activities that have meaning and purpose (p. 42)

Ability to explore, enjoy and describe patterns and relationships related to quantity, number, measurement, shape and space (p. 42).

Recognition that numbers can amuse, delight, comfort, illuminate, inform and excite (p. 42)

which are evident when Young children:

have opportunities to learn numeric symbols and to use mathematical concepts and processes, such as volume, quantity, measurement, classifying, matching and pattern recognition (p. 44) … using such strategies as setting and solving problems, looking for patterns, classifying, guessing, using trial and error, observing, planning, comparing, explaining, engaging in reflective discussion and listening to stories (p. 47)

Young children are:

encouraged to use trial and error to find solutions to problems and to use previous experience as a basis for trying out alternative strategies. They are encouraged to give reasons for their choices and to argue logically (p. 49)

The mathematics framework of *Te Kākano* can be helpful to describe valued learning for four year old children. The strands of *Te Kākano* are described as purposeful activities, including playing – takaro, designing, tinkering, inventing – hanga, pattern "sniffing" – tauira, measuring – inenga, understanding symbols and representing – whakamarama tohu, noticing, recognising, and constructing relationships – nga panga/hononga, positioning/classifying/being systematic – whakatakotoranga/whakaropu, reasoning and comparing and using data – whakaaro whaitake, visualising and imagining – tukua nga whakaaro pangarua, locating – kimi/rapu, positioning/grouping/classifying – takotoranga/whaka ropu, estimating and predicting – matapae, and calculating and counting – tataihia kautehia. These activities are closely related to valued mathematical practices, set in meaningful contexts of children's interest and play, in either individual or shared activity. Understandably there is a range of prior experience and learning prior to school entry due to the diversity of family and community experiences as well as the wide range of early childhood settings.

Important mathematical learning for four-year-olds includes the following. Bold indicates the most important material important for progress in mathematics and statistics learning:

- Investigating patterns and collections of objects in purposeful/play contexts, leading to **forming patterns, extending, checking and repairing patterns**. These might be repeating patterns such as beading contexts, objects in repeated rows, music, dance, etc, recognising and repeating others' patterns.
- In collections of objects, **organising, categorising and ordering might lead to counting to find out 'how many' objects** in different groupings and understanding

more and less in relation to how many in sets of objects. A range of counting processes might be used including counting in ones, pairing and counting in twos etc.

- Related processes are comparing and noticing equal amounts/quantities, recognising numbers on dice and dominoes, leading to recognising visual patterns for numbers to 6 (early subitising). Estimating using a 'thinking' guess.
- Using their **home language for counting might occur during a counting process, within contexts of stories, rhymes and songs, and ritualised counting up to 5 (or more)**. Counting might be in a process of allocating objects for sharing or fairness. The number 5 is a significant benchmark number as four-year-olds approach their 5th birthday.
- Spatial patterns involve using two dimensional and three-dimensional space. Further spatial knowledge is evident in building/constructing models for purposeful play including fantasy play. **Using 3D materials such as blocks for constructions and using trial and error to position objects in space. Exploring with 2D shapes such as in paper folding and cutting, knowing names for simple shapes, moving, flipping and placing together shapes**, and using drawings to represent space. **These might be drawings of models, from stories, or maps of a built setting (the ece centre) or of a journey (based on a familiar walk or other journey)**.
- Comparing as a means of making sense of our world. Exploring quantities such as water, weight, height and other lengths, space etc. **Using language that describes sameness and difference and starting to see smaller lengths (or shapes) within larger lengths (or shapes).**

(ii) An overview of Year 0-6 mathematics and statistics decisions

Our intention is to signpost transition and continuity across sectors and this transition and continuity is in mind as informing our rationale for waiting until ākonga have been at primary school for two terms before identifying signposts. As an overall principle, our young ākonga are capable of more than we are asking them for at present. In determining a year-by-year approach for thinking about mathematics and statistics content, suitable key changes in pace and emphasis became apparent. These are outlined in detail in the year-by-year templates, and summarised here:

- We should 'respond sooner' by beginning after two terms at school rather than waiting for a year. This is intelligent noticing of what ākonga know so their progress can be enhanced if need be, rather than a labelling approach. It also enables two terms of transitioning time into a new school context, and can accommodate the different times of the school year that akonga begin primary school.
- The suggestions emphasise patterning and structure with a reduced emphasis on counting as the key route to arithmetic and number understanding. This is not a 'pendulum swing', it is acknowledging that counting is important and relevant, but research and our experience with counting-based number interventions, suggest it is not enough for providing secure foundations for mathematics and statistics learning. Ākonga need counting and pattern and structure.

- The idea of relative magnitude of numbers is key. Subitising and comparing numbers of things without counting are part of developing a sound understanding of relative magnitude.
- Ākonga who make the most progress in mathematics and statistics see links among the things they learn, recognising when something they know applies to a new situation - seeing how place value links to metric measurement for example - or recognising a 'rule' such as the commutative property and understanding it will always apply. We can help all ākonga by teaching such links as well as the skills and knowledge. Some of the signposts embed this idea – that ākonga need to learn relationships among the ideas, seeing patterns or applications across mathematical and statistical contexts. Ākonga need to able to use these features of the number system so that they can generate processes and solutions. They need to know how the number system 'works' – the structure of the number system, and the patterned way of thinking about place value and relationships between numbers, which is all part of having number sense. This idea could be further promoted in the way the curriculum itself is structured and expressed.
- The signposts suggest we start (and then expect) some things earlier than some current practice, including: grouping, sharing and the conceptual roots of multiplication, fractions, decimals and percentages, equality and functional thinking. They also provide guidance on when ākonga should have particular tools in their mathematics and statistics toolkit. In particular, ākonga should be using multiplicative reasoning across all their mathematics and statistics work by the end of Year 6. The evolution of this progress can be traced through the signposts which also suggest earlier competence with addition and subtraction, while making it clear that robust multiplicative reasoning does not grow out of additive reasoning, it needs to be encouraged in itself, hence the inclusion of signposts about arrays and fluency with multiples and factors.
- The indicators recognise the significance of spatial reasoning for progress and understanding in mathematics. Spatial reasoning seems to contribute to seeing pattern and structure in numbers and to developing ideas about relative magnitude using powerful spatial models such as number lines or place value blocks. There is evidence that spatial reasoning can be improved by teaching (i.e., it is not an innate, fixed ability). Therefore, attention is paid in the signposts to aspects of spatial reasoning and representations of number using that use space (such as arrays, number lines, growing patterns that use shape).

Our aim in the templates was to use language accessible to ākonga, whānau, and teachers.

I bring useful knowledge and understanding to my mathematics and statistics learning.

After two terms, the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is refining and linking ideas of relative magnitude and subitising to the formal counting system
Algebra is making patterns, and seeing patterns in things, including shapes and colours
Geometry is making, seeing and talking about patterns in shapes
Statistics and Probability is sorting things into categories that show something about the collection of things

After two terms at school we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

<table>
<tr><th></th><th>Number and Measurement</th><th>Algebra</th><th>Geometry</th><th>Statistics and Probability</th></tr>
<tr><td>Problems and models</td><td rowspan="3">⌛ Subitise to five using different ordered arrangements (tens frames, cards, dice)
⌛ Count and enumerate sets to at least 10
⌛ Compare two amounts (without counting) and say which is smaller or larger
⌛ Choose two cards that show 'the same' from a set with different pictorial representations of numbers (organised and not organised) and numerals, up to 5</td><td rowspan="3">♒ Continue a two-element repeating pattern</td><td rowspan="3">⌘ Copy a 2-D pattern with a line of symmetry, using a few simple shape blocks</td><td rowspan="3">⧫ Sort a pile of pattern blocks or cubes into colours and say something about what they've found</td></tr>
<tr><td>Reason and critique</td></tr>
<tr><td>Patterns and structure</td></tr>
</table>

In mathematics and statistics I am looking for patterns and the way things are related to each other.

In Year 1 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is understanding how amounts, numerals and number names are patterned and linked together, up to 100
Algebra is recognising what 'the same' means with amounts and shapes and looking for patterns in what they are learning
Geometry is being able to see and describe patterns that use simple symmetry and shapes
Statistics and Probability is using comparison to discuss investigations and their outcomes, including the language of chance

In Year 1 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique** **Patterns and structure**	⌛ **Start counting at any two digit number and count up from there** ⌛ Skip count in 2s, 5s and 10s ⌛ **Use subitising to identify groups within sets and find totals to 20** ⌛ Share amounts out fairly ⌛ Find ½ and ¼ of a group of objects or a region ⌛ **Join and separate groups and numbers** ⌛ **Match numerals with amounts and number names to 20** ⌛ **Order numerals and pictures of sets according to their relative magnitude (to 100 at least)** ⌛ **Order and compare objects by length, by direct comparison**	≋ **Make repeating patterns and explain the pattern** ≋ Identify regularities and patterns within objects/pictures/arrays/patterns and in sequences of numbers ≋ **Identify and describe patterns in numbers to 100** ≋ **Identify things that are 'the same' (amounts, shapes in different orientations)**	⌘ **Create patterns using symmetry and explain their regularities** ⌘ Choose a shape from a group that matches a given description	⧫ **Discussing and telling and re-telling the stories of our investigations, using language of comparison** ⧫ Discussing and re-telling stories of their own explorations of statistical or probability contexts, and/or displays/drawings ⧫ Talking about things that might happen and possible outcomes ⧫ Asking questions and sorting into categories/groups ⧫ Counting and displaying data using people or objects, or drawings and pictographs

I can use pattern, number sense and simple statistical investigations to solve problems and learn new ideas.

In Year 2 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is developing a sound understanding of relative magnitude of numbers and begin to join, separate, compare and group to solve problems using this understanding as base

Algebra is seeking and describing patterns in the number system

Geometry is recognise, describe, and draw different shapes, and being able to see and talk about the effects of turns (rotation) and flips (reflections) on shapes

Statistics and Probability is working with others to pose a question, collect category data, sort it, discuss what was found and understand that not all responses or outcomes were equally likely

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 2 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	**Start counting at any two digit number and count back from there** Skip count forwards and backwards in 2, 5, 3 and 10 **Add and subtract amounts - two digit numbers +/- single digit numbers** **Know and describe the patterns in basic addition and subtraction facts** Use patterns to extend addition and subtraction e.g., given 21+3 = 24, what is 31+3 etc **Split and recombine numbers in groups e.g., split 20 into 10 lots of 2** Find ½, ¼, ⅛, ⅓, ⅙ of a set or region **Match numbers with representations of amounts and names to 100** **Order amounts according to relative magnitude:**	**Identify and describe patterns in numbers to 1000** Create a growing pattern using materials	⌘ **Recreate a pattern they've been shown that uses symmetry and rotation** ⌘ **Solve spatial puzzles involving missing pieces and rotations** ⌘ **Rotate 2-D shapes mentally and physically, knowing what the resulting shape looks like** ⌘ Sort objects by their appearance. ⌘ Give and follow instructions for movement that involve distances, directions, and half or quarter turns. ⌘ Describe their position relative to a person or object. ⌘ Communicate and record the results of translations, reflections,	⧫ **Asking questions, collecting data and sorting into multiple categories** ⧫ Counting, comparing (what's the same and what's different), and discussing ⧫ Displaying data in simple tables and graphs e.g., pictographs, bar charts, simple stem and leaf ⧫ **Discussing and answering our initial questions, telling the stories of our investigations** ⧫ Listening to descriptions of others' investigations/simple displays. Counting, comparing (what's the same and what's different), and discussing

Patterns and structure	**whole numbers to 1000 and 1/2s and 1/4s** **Put numbers on a number line to show relative magnitude** **Compare two amounts to 20 and say which is bigger and by how much without counting** **Order and compare objects by length, by counting whole numbers of units**		and rotations on plane shapes.	category data, using language of comparison. ⧫ Talk about what the outcomes of something might be and how predictable they might be

Whakataukī/Motto/Kīwaha/Saying **Year 3 Pāngarau Mathematics and Statistics Tauanga**

I understand how the place value structure of the number system works and can use this in exploring measurement and statistical situations.

In Year 3 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is understanding the place value system, and relate to number and measurement contexts
Algebra is knowing '=' means 'the same as' and being able to use it to show equalities
Geometry is moving, combining and drawing patterns of combinations of 2D shapes, and extend to moving 3-D objects in space
Statistics and Probability is working with more than one variable that describes a data set that they have collected and making sense of others' descriptions

In Year 3 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models	**Have a useable and flexible understanding of the place value structure of whole numbers, explain the patterns and regularities that make it work** **Skip count forwards and backwards in multiples** **Recall basic addition and subtraction facts to solve problems** Use a number line to record skip counting, place numbers and fractions in order of relative magnitude, create and use a ruler	Make, extend and describe patterns that grow **Use = to show that things are the same**	♦ **Make, copy and describe patterns that involve symmetry and rotation** ♦ **Solve 2-D and 3-D spatial puzzles with missing pieces**	♦ **Asking questions and making a plan, gathering data, sorting data and explaining their reasoning** ♦ Record results using tally marks or other. Organising and displaying data using tables or simple graphs such as pictographs, strip, bar charts
Reason and critique	**Group amounts using 10s and 1s and operate with 10s as a unit** **Know complementary numbers that add to 10** **Share and find the number of shares and how much is in each share** See, describe and use the structure of an array in rows and columns Find equivalent fractions for ½ **Multiply single digit numbers and model the process using array**		♦ **Find or draw a shape that 'belongs' in a group of shapes** ♦ **Know position words and describe where things are in**	♦ Begin to collect measurement data ♦ Discussing our initial questions, revising our planning and communicating our findings ♦ **Interpreting visual information in others' statistical displays of data such as graphs**

Patterns and structure	**models and other spatial models** **Compare any two amounts and say which is bigger and by how much** **Describe the patterns in families of facts** **Identify different characteristics of shapes, events and objects that can be measured** Know ways to approach measuring length, area, volume and capacity, weight (mass), turn (angle), temperature, and time Tell the time on analogue and digital clocks		**relation to each other**	⧫ Agreeing or disagreeing with statements made about data displays, and explaining with reasons. ⧫ Have vocabulary to discuss likelihood in everyday situations

I understand that there are patterns in operations on numbers and data displays.

In Year 4 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

- Number and Measurement is developing consistently reliable methods for addition and subtraction, and beginning to understand rational numbers (including decimals and percentages) (how they show parts of a number or region or a amount)
- Algebra is developing a solid understanding of how addition and subtraction on whole numbers work - what they are asking you to do to numbers and what some of the underlying rules are about how they work
- Geometry is knowing the spatial features that shapes and objects have and using these to sort and recognise shapes and objects in different places and orientations
- Statistics and Probability is working with more than one variable that describes a data set collected by others and exploring the likelihood of particular outcomes

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 4 we act and think mathematically and statistically in these ways:

- Make sense of and solve problems and use models, with perseverance
- Reason and argue mathematically and statistically, with precision, and critique others' reasoning
- Look for, make use of, and communicate about patterns, regularity, and structure

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	**Have a reliable and accurate method for solving any addition and subtraction problem with whole numbers** **Use principles and patterns to solve multiplication and division problems of different types with whole numbers, including measurement models of division** **Put unit fractions on a number line** **Understand non-unit fractions as parts of a region** Express a proportion using a percentage Use patterns and basic facts knowledge to find common multiples and factors of numbers Use patterns and basic facts knowledge to find equivalent fractions	**Use the inverse property to solve addition, subtraction, multiplication and division problems** **Use the commutative property to solve addition and multiplication problems** **Use 0 as the additive identity** **Use 1 as the multiplicative identity** **Use = to show that two**	⌘ **Sort objects by their spatial features, with justification** ⌘ Identify and describe the plane shapes found in objects ⌘ Create and use simple maps to show position and direction ⌘ Describe different views and pathways from locations on a map. ⌘ **Predict and communicate the results of translations, reflections, and**	⧫ Asking questions and making a plan, gathering and sorting data, making decisions about data. **Organising and displaying data using tables or graphs –** expanding range of possible displays, and explaining which display is best for their investigation, with reasoning (dot plots, stem and leaf, bar or pie charts, etc). Beginning to interpret the patterns in and shape of the data distribution ⧫ Discussing initial investigation questions, revising our planning and communicating our findings

Patterns and structure	**Use relationships between numbers to understand and solve problems e.g., the relationships among 2,3 and 6** Work with tenths as an extension of place value knowledge Use appropriate devices to measure length, mass, area and volume	**expressions are the same**	**rotations on plane shapes**	⧫ Interpreting visual information in others' statistical displays and critically consider statements about any patterns in the data ⧫ **Begin to identify advantages and disadvantages of simple graphs –** bar, pic and stem and leaf, with reasons ⧫ Investigate simple situations that involve elements of chance, recognising equal and different likelihoods and acknowledging uncertainty

I am shifting to a multiplication-based understanding of how numbers work and use this to understand and express probability.

In Year 5 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

> Number and Measurement is establishing a method for multiplication and using place value, fraction and decimal understanding in measurement
> Algebra is using patterns to make predictions, and using inequalities to express relationships between numbers and expressions
> Geometry is describing and modelling equivalence and difference in the context of shapes as well as numbers
> Statistics and Probability is thinking critically about category data displays created by themselves and others

In Year 5 we act and think mathematically and statistically in these ways:

> Make sense of and solve problems and use models, with perseverance
> Reason and argue mathematically and statistically, with precision, and critique others' reasoning
> Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique** **Patterns and structure**	**Have a reliable and efficient, accurate method to solve multiplication problems, making sensible choices about methods (hand or machine)** **Know basic multiplication facts and use them to solve more sophisticated problems** Put unit and non-unit fractions on a number line Put percentages on a number line Work with decimals to 100ths as an extension of PV knowledge and percentages, and measurement **Solve measurement problems using metric measurements for length and mass**	**Continue a sequential number pattern and say why it continues that way** **Create and continue spatial patterns** **Use inequalities to express relationships between amounts and expressions**	⌘ **Use coordinates to define locations, and different ways of describing location and directions (compass points etc)** ⌘ **Be able to find invariant properties/equivalence between simple shapes and explain these using appropriate language** ⌘ **Identify which shape is a rotation or reflection of a specified shape from a range of choices**	⧫ Using existing data to expand range of display representations such as dot plots, back-to-back stem and leaf etc. Using digital technology such as spreadsheets to record data and generate data displays. **Identify possible advantages and disadvantages of different kinds of displays** ⧫ Using stem and leaf graphs to find the median and mode ⧫ Communicating interpretations of the data and posing more questions for investigation ⧫ **Investigating existing data – that are displayed in different ways – tables and statistical graphs** ⧫ **Asking relevant questions and making sense of the data** ⧫ Model the outcomes of a chance situation systematically

I am relating multiplication to place value, rational numbers, measurement systems, number patterns, geometrical patterns and relationships, data displays, probabilities, and different types of problem situations.

In Year 6 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is using multiplicative approaches to understanding place value, rational numbers and solving problems
Algebra is knowing how the four operations 'work' and beginning to use functional thinking to understand and describe two-variable patterns
Geometry is using formal language of geometrical properties to describe shapes, transformations and spatial positions
Statistics and Probability is collecting and using time series data as well as summary and comparison data, and being able to model the likelihood of outcomes in controlled probability situations (such as rolling two dice)

In Year 6 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	Solve problems involving rates, multiplicative comparison, part-whole multiplicative relationships, cartesian products and arrays, quotative and partitive division on whole numbers **Approach multiplicative problems using multiplicative reasoning, not additive reasoning** Understand improper fractions and put them on a number line **Understand fractions as regional models, parts of amounts, products of division** **Put any whole number, fraction, decimals or percentage on a number line to show relative magnitude** **Understand the PV system for whole and**	**Explain what's 'allowed' in each operation: commutative, distributive, associative and inverse properties** **Identify equality and equivalence in numbers, shapes and shape properties, fractions, rational number representations** **Use a table they are given to organise an explanation of a pattern that relates the term of the pattern to the pattern sequence**	⌘ Represent objects with drawings and models ⌘ **Classify plane shapes and prisms by their spatial features** ⌘ **Use a co-ordinate system or the language of direction and distance to specify locations and describe paths** ⌘ **Describe the transformations (reflection, rotation, translation, or enlargement) that**	⧫ Designing and carrying out all phases of a statistical investigation. Making decisions about data collection, and the organising and displaying data. Link reasons for decisions with parts of the statical investigation process such as the inquiry questions or the kinds of data. Choosing appropriate displays of data with reasoning, and include displays for multivariate and simple time series ⧫ **Looking for patterns in the data, identifying the spread of data, and identifying and discussing patterns and trends in the data, (spread and outliers) and communicating our findings in text and visually** ⧫ **Investigating existing data, including multivariate data – that are displayed**

	decimal numbers ⌛ Convert between metric measurement amounts, especially length: cm, mm, m, km ⌛ **Understand number as continuous** ⌛ Solve measurement problems using metric measurements for length, area, volume and capacity, weight (mass), ⌛ Find areas of rectangles and volumes of cuboids by applying multiplication. ⌛ **Develop benchmarks for conversion between fractions, decimals and percentages (equivalents for half, quarter, third, fifth, three quarters, tenth)**	**(functional rather than recursive thinking)**	**have mapped one object onto another**	**in different ways – tables, graphically or other such as tree diagrams** ⧫ **Asking relevant questions, making sense of the data, and explaining choices of displays for different contexts** ⧫ Communicating interpretations of the data and posing more questions for investigation ⧫ Investigate simple situations that involve elements of chance by comparing experimental results with expectations from models of all the outcomes, acknowledging that samples vary
Patterns and structure				

(iii) Years 7-10

The signposts for Years 7 and 8 have been aimed at encapsulating much of the current Level 4, but assuming that there has been stronger progress up to Year 6 than in current expectations. Years 9 and 10 content was informed using the current level 5 curriculum guidelines and relevant work in overseas curricula. Focus was given to providing examples of ways mathematical processes and digital technology and purposeful use of mathematics can be woven into statements of required content.

I can use my multiplicative reasoning, pattern recognition and number sense to solve problems, and to investigate statistical contexts and relationships, and critique solutions and conclusions.

In Year 7 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is solving problems using fractions, decimals, and percentages
Algebra is using tables, graphs and rules (words or numbers) to discover and describe sequential patterns
Geometry is using specific geometrical language that enables precision in describing and replicating shapes and design
Statistics and Probability is working with relationships in and through data, describing the shape of the data, and using continuous numerical variables

In Year 7 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

<table>
<tr><th></th><th>Number and Measurement</th><th>Algebra</th><th>Geometry</th><th>Statistics and Probability</th></tr>
<tr><td>Problems and models</td>
<td rowspan="3">⌛ Recognise what operations to use in a problem involving whole numbers and use them accurately to find an answer. Use checking strategies
⌛ Use a range of estimation strategies for whole number situations
⌛ Use fractions, decimals and percentages in contexts needing simple proportional reasoning
⌛ Know equivalent forms of the same simple fractions
⌛ Know the equivalent representations and simple conversions for fractions, decimals and percentages
⌛ Find percentages of amounts, including using calculator
⌛ Use calculation strategies to find perimeters and areas of simple shapes – rectangles including squares
⌛ Read scales of measuring instruments using knowledge of place value and decimals with appropriate measurement units</td>
<td rowspan="3">♒ Recognise and using patterns in the number system to solve problems (odds and evens, multiplication patterns etc)
♒ Express number relationships with the equals sign representing equivalence on each side, for simple combinations of operations
♒ Represent number patterns in multiple ways – sequence, with words, models, tables, and simple graphs
♒ Recognise and describe how to find a term in a pattern using words and numbers</td>
<td rowspan="3">⌘ Visualise, describe and make drawings and models of two and three dimensional shapes.
⌘ Explore nets for cubes and cuboids
⌘ Know and use geometrical properties to describe and classify two dimensional and three dimensional shapes, with reasoning
⌘ Know more complex shapes such as composite shapes and convex shapes
⌘ Making sense of our environment using bearings, coordinates, distances and simple grid references
⌘ Visualise, draw and describe transformations of reflection, translation, rotation and enlargement,</td>
<td rowspan="3">⧫ Analyse the shape of the data distribution and identify the range, and measures of centre – median, mean and mode
⧫ Communicate interpretations of data and pose questions for investigation
⧫ Investigate existing data, including multivariate data – displayed in different ways – e.g., using tables and graphs
⧫ Make sense of a greater range of presentations of statistical data from the local community or within the media (such as displays , interpretations and statements). Begin to ask</td></tr>
<tr><td>Reason and critique</td></tr>
<tr><td>Patterns and structure</td></tr>
</table>

	⌛ Begin to consider precision of measurement scales		with increasingly complex designs and patterns	critical questions of the presentation of data and findings

Whakataukī/Motto/Kīwaha/Saying **Year 8 Pāngarau Mathematics and Statistics Tauanga**

I can reason proportionally using relationships and properties of number, shape and data.

In Year 8 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is choosing appropriate calculations and number representations to solve whole number, rational number and integer problems
Algebra is representing and explaining mathematical relationships in different ways, including using letters to represent variables and unknowns
Geometry is understanding features of shapes and space and moving between different models/representations of 2 and 3D shapes and objects
Statistics and Probability is engaging in all phases of a simple statistical investigation involving comparison or relationship situations, using and critiquing their own data sets and data sets from others, and be able to explain why the results of a probability experiment might not be the same as a model would predict

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 8 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ Rational numbers can be represented and operated on in a variety of ways to solve problems. ⌛ Add/sub/mult/div on whole numbers, fractions, decimals, integers (may have to specify??) ⌛ **Choosing the best calculation process for different numbers and/or different contexts. Consolidating a range of checking processes and recognising when estimation should be used** ⌛ Competently use properties of the number system in different situations – e.g., distributive, associative, identity properties	♒ Be able to use symbols to express a function rule describing a pattern, and to graph this relationship ♒ **Using tables, graphs and equations to represent a linear number pattern. Explain the mathematical relationship and the advantages and features of each representation** ♒ Use letters to represent unknowns and variables in expressions (letter is the "number of' where students choose the letter with reasons) ♒ Using one of the above	⌘ **Consolidating knowledge of transformations – reflection with a range of mirror lines/axis of symmetry. Transformations of more complex shapes such as composite shapes** ⌘ Use proportional reasoning to understand geometrical relationships such as scaling (enlargement and reduction) ⌘ **Designing and testing ways of representing 3D shapes with 2D models (nets and plan views). Increasing confidence with the relationships between faces, edges and**	⧫ **Plan all phases of a statistical investigation, making decisions about variables and data collection methods. Organising and displaying data using tables or graphs – multivariate and simple time series** ⧫ **Using technology where appropriate for organising and displaying data** ⧫ **Identifying patterns in the data including spread and patterns at the centre, and communicating our findings. Visually, verbally and with written explanations.** ⧫ Expanding the range of ways that statistics is presented in everyday contexts. Looking for statistically sound

Patterns and structure	Converting between fraction, decimal and percentage representations in purposeful contexts. Solve (simple) ratio problems using proportional reasoning Put integers on a number line Use appropriate scales, devices, and metric units for length, area, volume and capacity, weight (mass), temperature, angle, and time **Confidently convert between units showing a place value understanding for large numbers up to one million, and small numbers to thousandth. Relate to everyday uses of measurements and their units**	representations to answer 'what if' questions **Writing mathematical equations in different ways showing an understanding of the equals sign and a balancing of each side (for equations that balance)**	**corners for 3D for a range of prisms** ⌘ Knowing the 8 points of the compass and being able to read maps, interpreting direction and distance ⌘ Exploring relationships between area of a rectangle and the area of parallelogram, triangle, trapezium, and kite, by folding, or cutting. (not by formulas) explaining the relationships in words and visually	statements based on the data presented. ⧫ Also consider if there might be anything missing or alternative findings from the data. Probability ⧫ Investigating a greater range of chance experiments such as two stage situations, with repeated trials, using technology or other methods to generate a very large number of trials. Exploring the concept of independent events in the context of their probability experiment. Making statements about their conclusions, with reasoning, including about the theoretical probability.

I can use strong content knowledge to select appropriate solution processes in a range of mathematics and statistics contexts.

In Year 9 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

> Number and Measurement is developing confidence in their use of adding, subtracting, multiplying and dividing proportions, and in estimating and measuring
> Algebra is making and solving algebraic equations and working with linear functions to solve practical problems
> Geometry is using technology and geometrical relationships to resolve real world problems involving 2D shapes, scale and symmetry
> Statistics and Probability is understanding that variation and chance affect results of statistical investigations and probabilities of events happening

In Year 9 we act and think mathematically and statistically in these ways:

> Make sense of and solve problems and use models, with perseverance
> Reason and argue mathematically and statistically, with precision, and critique others' reasoning
> Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ Know fraction, decimal and percentage conversions, and flexibly apply to different contexts ⌛ **Confidently add and subtract fractions with like and unlike denominators** ⌛ **Find and use ratios to solve real life problems** ⌛ **Consolidate calculation processes with integers including many factor multiplication of negative integers** ⌛ **Expand use of rounding, significant figures, and standard form to more complex situations** ⌛ Exploring how whole numbers can be represented by prime numbers, by powers	**Be able to analyse linear patterns using tables, diagrams, and graphs, and find the general rule of linear patterns and equations of lines, and use the rule/equation to find unknown terms and make predictions of real life situations involving linear patterns** Be able to expand and factorise brackets in linear equations and identify and collect like terms and describe the mathematical properties that enable this	⌘ **Understand and use Pythagoras' rule to find lengths of side lengths of right angled triangles** ⌘ Use terms for and properties of angles on parallel lines and similar figures to draw and justify conclusions ⌘ Create accurate nets for simple polyhedra for purpose ⌘ **Compare properties of 3D shapes including in relation to their purposes of these shapes in everyday objects** ⌘ **Make and use scale drawings and**	Statistical Investigations ⧫ **Plan and carry out surveys using the statistical enquiry cycle to explore summary type questions** -determining appropriate variables and measures; -considering sources of variation; -gathering data; -using multiple displays, and re-categorising data to find patterns, variations, relationships, and trends in multivariate data sets;

Patterns and structure	of 2 etc. Generating different sequences of numbers ⌛ **Estimate, measure, calculate and use formulae for finding perimeters and areas of simple 2D shapes and combinations of these – both for abstract shapes and in real life contexts (parallelograms, quadrilaterals, triangles,)** ⌛ Exploring pi and using pi in circle problems. Find area and perimeter of composite shapes that involve circles or parts of circles. ⌛ Estimate, measure, calculate and use surface areas of 3D shapes and combinations of these for purpose (prisms, cuboids, sphere, cones, pyramids) ⌛ **Discuss and take into account accuracy of measurement devices and measurements made using these and consider implications of measurement errors in real life situations**	**Be able to confidently define terms and write equations that represent simple everyday linear situations, solve for unknown values, interpret the solution in light of the context, and justify and explain their mathematical thinking and decisions (e.g., perimeter and area of a new sports field)** Horizon knowledge: be able to show inequalities on number lines and graphs	⌘ **Represent 3d shapes in 2D using isometric paper and other methods** ⌘ Provide and use directions to follow a path using bearings and distances ⌘ Discuss properties of transformations, including congruence and invariance, and how these properties help us understand and use examples of transformations in everyday life ⌘ Discuss and compare properties of everyday objects in relation to how reflection, rotation, translation enable specific uses and artistic features	-comparing sample distributions visually, using measures of centre, spread, and proportion; -presenting a report of findings Probability ⧫ **Evaluate probability activities undertaken by others, including data collection methods, choice of measures, and validity of findings** ⧫ **Calculate probabilities, using fractions, percentages, and ratios**

Whakataukī/Motto/Kīwaha/Saying **Year 10 Pāngarau Mathematics and Statistics Tauanga**

I can use proportional reasoning in number, algebra, measurement, geometrical, and statistical contexts.

In Year 10 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is being able to select from and use a range of number and measurement tools and strategies in number, measurement, algebraic, geometric and statistical contexts
Algebra is understanding relationships between information in tables, equations, and graphs for simple patterns and functions
Geometry is using technology, including geometry software, and geometrical relationships to understand and resolve real world problems involving 2D and 3D shapes, objects and symmetry
Statistics and Probability is understanding how the design of an experiment, data collection, sampling and analysis impact on findings of an investigation

In Year 10 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

<table>
<tr><th></th><th>Number and Measurement</th><th>Algebra</th><th>Geometry</th><th>Statistics and Probability</th></tr>
<tr><td>Problems and models</td>
<td rowspan="3">⌛ Investigating percentage change related to different contexts and critically discuss solutions, and implications for the context.
⌛ Consolidate processes for finding and using rates and ratios.
⌛ Explore multiplication of powers, and relate to exponential growth contexts. Also explore negative exponents.
⌛ Move flexibly between squares and square roots, for a greater range of numbers, including decimals and fractions.
⌛ Estimate, measure, calculate and use volumes of 3D shapes and combinations of these for purpose (prisms, cuboids, sphere, cones, pyramids)
⌛ Estimate, measure, calculate and use simple rates (e.g., distance covered in time taken, rate of flow or pour of</td>
<td rowspan="3">♒ Explore with and without software and discuss features of graphs of linear, quadratic, power and exponential functions to understand how gradient, intercepts, constants, turning points and sense are seen in graphs and function equations and to understand and make predictions about real life situations
♒ Be able to expand and factorise brackets in quadratic equations in order to solve and display quadratics</td>
<td rowspan="3">⌘ Using trigonometric ratios with right angled triangles to find side lengths and angles
⌘ Deduce and use properties interior and exterior angles of polygons and use them to argue/justify responses
⌘ Use coordinate planes, including paper based and software based coordinates, to investigate and describe properties of transformations, including congruence and invariance
⌘ Connect three-dimensional solids with different two-dimensional representations
⌘ Construct and describe simple loci for functions, relations, and other geometrical and real life purposes</td>
<td rowspan="3">Statistics Investigations

♦ Plan and carry out experiments using the statistical enquiry cycle to explore relational and comparative type questions
-determining appropriate variables and measures;
-considering sources of variation;
-gathering and cleaning data;
-using multiple displays, and re-categorising data to find patterns, variations, relationships, and trends in multivariate data sets;
-comparing sample distributions visually, using measures of centre, spread, and proportion;
-presenting a report of findings

Statistical Literacy

♦ Evaluate statistical investigations undertaken by others, including data collection methods, choice of measures, and validity of findings</td></tr>
<tr><td>Reason and critique</td></tr>
<tr><td>Patterns and structure</td></tr>
</table>

	liquid, pace of speech in different languages) ⌛ Horizon knowledge: explore, describe, and consider reasons for the shapes of cross sections of three dimensional shapes	♒ Be able to change the subject of a formula ♒ Confidently add and subtract simple algebraic fractions	⌘ Use vectors to describe movements and combinations of movements (e.g, addition/subtraction of vectors and multiplication of vectors by numbers)	Probability ♦ Compare and describe the variation between theoretical and experimental distributions in situations that involve elements of chance

(iv) Years 11-13

The signposts for year-by-year content for mathematics and statistics in years 11-13 are largely drawn from current practice. Deep consideration of suitable changes from this were not possible within the timeframe of this report preparation. Again, focus was given to providing examples of ways mathematical processes and digital technology and purposeful use of mathematics and statistics can be woven into statements of required content.

Recommendation:

- For Years 11-13, we strongly advocate for further urgent work on year-by-year signposting of mathematics and statistics content using a small team/s of experts working with an advisory/consulting group in an iterative consultative development process. Membership of these groups should include, for each of mathematics and statistics: at least one mathematics/statistics teacher, at least one mathematics/statistics university lecturer (with differing priorities for Year 11-13 curriculum content and emphasis), and at least one mathematics education researcher.

I can see and use links between different areas of mathematics and statistics.

In Year 11 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is being able to confidently use all of the number and measurement tools and understandings needed for being a numerate citizen and for continued learning in mathematics and statistics
Algebra is using equations, rules and graphs to explore real situations, solve problems and make predictions
Geometry is selecting and using geometrical tools to investigate geometrical situations involving shape, position and symmetry across measurement, geometry and algebraic contexts
Statistics and Probability is being aware of and able to critique chance and data situations relevant to important issues in their everyday lives and make decisions based on this awareness

In Year 11 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ **Use multiple operations to interrogate and resolve real problems involving real numbers** ⌛ Solve context-based number problems using direct and inverse relations and powers with integers and fractions ⌛ Define and use irrational numbers (e.g., pi, square root of 2) ⌛ Investigate situations involving compound rates to draw conclusions ⌛ Use numerical strategies to find optimum values of functions ⌛ **Measure at a level of precision appropriate to the task and determine, explain and justify measurement accuracy (including accuracy of measuring devices, accuracy of derived measures, significant figures)** ⌛ Select and apply formulae relating to simple three-dimensional	♒ **Define variables, form and solve linear equations and inequations, quadratic and simple exponential equations, and simultaneous equations with two unknowns for purpose** ♒ Use digital tools to investigate, describe and visualise transformations to functions caused by adding or subtracting a constant to the function and within the function and by multiplying the function by a constant ♒ **With and without digital tools, use graphs, tables, and equations of linear, quadratic, and simple exponential relationships found in number and spatial patterns and in everyday contexts to explore situations and issues (including**	⌘ Deduce, apply and communicate using angle properties related to circles ⌘ Recognise when shapes are similar and use proportional reasoning to find unknown lengths in context-based problems ⌘ **Apply trigonometric ratios and Pythagoras' theorem in two and three dimensions in problem solving and modelling situations** ⌘ Use a co-ordinate plane or map to identify and use points in common and areas contained by two or more loci ⌘ **Visualise, compare and apply single and multiple transformations in art, mathematical and other real-world applications**	Statistical investigation ♦ Design and use a statistical investigation to integrate statistical and contextual knowledge to answer investigative questions important to people's lives and make informal inferences about populations from samples, justifying variables and measures used in the data collection phase and taking variation and uncertainty into account ♦ Use multiple displays to show and discuss features of the sample distributions in light of statistical and contextual information (e.g., trends, relationships between variables, differences within and between distributions) and potential implications of this information ♦ Use informal methods to compare populations using sample

	figures (including prisms, pyramids, cones, spheres) to find unknown values	**important societal issues) and make predictions**	⌘ Draw, describe and analyse symmetrical patterns using the transformations in them or used to create them	distributions (including reasoning about shift, overlap, sampling variability and sample size)
Patterns and structure	⌛ **Understand and use conversions between units to apply measurements to solve context-based problems**	♒ **Understand that the gradient of a graph gives the rate of change and use gradients to make predictions** ♒ **Use gradients and y-intercepts to create equations for real world situations** ♒ Generalise and be confident to apply the properties of operations with rational numbers, including the properties of exponents		Statistical literacy ⧫ **Interpret and evaluate informal everyday statistical reporting such as in advertisements and magazine articles in light of statistical processes and by relating the displays, statistics, processes, and probabilities used to the claims made** Probability ⧫ **Explore chance situations involving discrete random variables, understand the role sample size plays in estimating probabilities via experiment, record results and plot frequencies of outcomes and investigate probability distributions** ⧫ **Calculate probabilities in discrete situations and explain and justify whether theoretical chance outcomes are or are not equally likely**

I can use tools of mathematics and statistics to understanding and explore mathematical and real world problems.

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 12 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Mathematics is using multiple representations for an expanded range of mathematical expressions, and bringing together algebraic and geometric techniques to solve problems

Statistics and Probability is organising and displaying complex data sets and using statistical tools to critically interpret and report findings

In Year 12 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

	Mathematics	Statistics and Probability
Problems and models **Reason and critique** **Patterns and structure**	Patterns and relationships ≈ **Identify, describe and use arithmetic and geometric sequences and series to investigate mathematical and everyday patterns and make predictions with consideration of effects of real world variables** ≈ **Use digital tools to display the graphs of linear and non-linear functions and identify and discuss relationships between the structure of the functions with their graphs** (including exponential, logarithmic and trigonometric equations (range 0-2π)) ≈ Derive and use trigonometric relationships, including the sine and cosine rules, in two and three dimensions and right and non-right angles triangle, to calculate lengths and angles to help solve geometric problems, with justification and consideration of accuracy ≈ **Confidently use and manipulate exponents, surds and logarithms as functions of numbers and of unknowns** ≈ Design, choose and justify appropriate networks to find optimal solutions for practical situations ≈ Derive and use co-ordinate geometry techniques to find aspects of points and lines with justification Equations and expressions ≈ **Confidently manipulate rational, exponential, and logarithmic algebraic expressions** ≈ **Form and use linear, quadratic, and simple trigonometric equations with and without digital technology and with and without focus on real world applications, interpreting and verifying solutions** ≈ **Form and use digital technology to explore solutions of pairs of simultaneous equations, one of which may be non-linear for purpose** Calculus	Statistics investigations ♦ **Use statistical investigations (with experimental and existing data sets) to explore and report on an important societal issue, make informal estimated comparison intervals for population parameters, informal predictions, interpolations, and extrapolations, considering sampling variability and sample size effects, and discuss implications of findings and potential recommendations for action** ♦ Design a questionnaire specific to a given purpose and use random sampling techniques for data collection ♦ Use results from statistical investigations to explore and ascertain risk and relative risk and interpret and communicate about risk Statistical literacy ♦ **Interpret and evaluate formal statistical reporting such as in social polls and surveys in light of aspects of the statistical cycle and considering sampling and non-sampling errors and use interpretation and evaluation to make predictions and discuss implications and recommendations for action** Probability ♦ **Investigate chance situations involving continuous variables comparing theoretical distributions (e.g., normal distribution) with experimental distributions**

	♒ **Investigate and describe the relationships between the graphs of functions and between the graphs of functions and their gradient functions (including through use of first principles), with and without digital technology and with and without focus on real world applications** ♒ Apply differentiation and anti-differentiation techniques to polynomials	♦ Use tools such as probability tree diagrams, two way tables, simulations and technology to calculate probabilities ♦ **Use technology-based and physically-based experiments to estimate probabilities**

Whakataukī/Motto/Kīwaha/Saying **Year 13 Pāngarau Mathematics and Statistics Tauanga**

I can use a range of sophisticated mathematics and statistics strategies and tools for exploring important mathematical and real world problems.

In Year 13 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Mathematics is using digital and other tools to represent, compare and contrast, and generate an expanded range of mathematical relationships

Statistics and Probability is using experimental design principles and statistical tools and techniques to critically interpret, evaluate and report for complex data sets and for probability context

In Year 13 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Mathematics	**Statistics and Probability**
Problems and models **Reason and critique** **Patterns and structure**	Patterns and relationships ♒ Apply the geometry of conic sections in mathematical and real world contexts ♒ **Use digital technology to investigate, display and interpret graphs of functions with graphs of their inverse and/or reciprocal functions** ♒ Use permutations and combinations to find the count and for solving probability problems ♒ **Use digital technology to use curve fitting, log modelling, and linear programming techniques for purpose** ♒ Develop, use and justify the design of network diagrams to find optimal solutions, including critical paths Equations and expressions ♒ Confidently manipulate and use trigonometric expressions ♒ **Form and use trigonometric, polynomial, and other non-linear equations with and without digital technology and with and without focus on real world applications, interpreting and verifying solutions** ♒ Form and use digital technology to explore systems of simultaneous equations, including three linear equations and three variables, and interpret the solutions in context ♒ **Confidently manipulate complex numbers and present them graphically** Calculus ♒ **Understand, describe and use connections between functions and gradient functions** ♒ Identify and interpret discontinuities and limits of functions	Statistical investigations ♦ Use different data collection contexts using experimental design principles to integrate statistical and contextual knowledge to answer important investigative questions, use statistical models (e.g., including **linear regression for bivariate data and additive models for time-series data**), seeking explanations, and making predictions, make statistical inferences about populations or processes from samples (e.g., using bootstrapping or randomisation) to determine estimates, confidence intervals, forecasts, and strength of evidence, evaluating all stages of the cycle, and using this to discuss implications and recommendations ♦ **Explain sampling variability and sample size effects** Statistical literacy ♦ Make inferences from surveys and experiments on important social contexts determining estimates and confidence intervals for means, proportions, and differences, recognising the relevance of the central limit theorem and using methods such as resampling or randomisation to assess the strength of evidence ♦ **Evaluate a wide range of statistically based reports, including surveys and polls, experiments, and observational studies and critique causal-relationships claims and interpret margins of error** Probability ♦ Investigate chance situations using probability concepts and distributions including using concepts such as randomness, probabilities of combined events and mutually exclusive events, independence, conditional

	♒ **Choose and apply a variety of differentiation, integration, and anti-differentiation techniques to functions and relations, using both analytical and numerical methods for mathematical and real world problems, with and without digital technology, interpreting and verifying solutions** ♒ **Form and justify differential equations for modelling simple mathematical and real world situations and interpret the solutions**	probabilities and expected values and standard deviations of discrete random variables, and probability distributions including the Poisson, binomial and normal distributions ♦ **Model chance situations using discrete and continuous probability distributions** ♦ **Use probabilities from theoretical models and estimate probabilities using experiments**

Pedagogy - Important cross-discipline links

We have considered important cross-disciplinary links between mathematics and statistics and other disciplines in three ways:

- how suitable material from other disciplines can be reinforced within a mathematics teaching and learning programme (e.g., see initial approach for considering this in relation to te reo Māori, Appendix Two)
- how suitable material from mathematics and statistics can be reinforced within teaching and learning programmes of other disciplines
- how mathematics and statistics and other disciplines can be taught in integrated topics, (i.e., topics which are designed and taught to encompass teaching and learning from two or more disciplines).

(i) Supporting teaching of material from other disciplines within mathematics and statistics teaching and learning programmes

The main focus of mathematics and statistics teaching is developing mathematics and statistics learning. At times, there are good opportunities to support the development of understanding of material from other disciplines within this. Links with the teaching and learning of other disciplines are particularly useful when they help develop mathematical and statistical knowledge and understanding, awareness of the real world uses of mathematics and statistics and engagement with and motivation for learning mathematics and statistics. Making such links effectively and with integrity in teaching and learning programmes requires sound content and pedagogical content knowledge both of mathematics and statistics and the discipline being linked with.

Important opportunities for supporting learning of other disciplines within mathematics and statistics teaching and learning include:

- teaching and using te reo Māori related to the mathematics and/or statistics learning purpose (e.g., see Appendix Two)
- teaching using science investigations as contexts for reinforcing understanding of measurement and emphasising key ideas, such as accuracy, estimation, reading scales, use of rational numbers, use and interpret data display for category or measurement data, understanding that a sample may or may not represent the actual population characteristics and the usefulness of repeating procedures for increasing certainty
- teaching using a technology curriculum emphasis e.g., using measurement to help solve a design-based problems to meet specifications, using knowledge of position and transformation to programme simple paths
- teaching using a literacy and English curriculum emphasis, such as through using critical evaluation of claims made in persuasive texts and media using understanding of statistics and percentages

- reinforcing number and use and interpretation of data and probabilities within health-related contexts and the social sciences to critique claims and data displays, reflect on whose interests are served in how the numbers are presented, make judgements and explore and understand consequences
- reinforcing number and measurement ideas through examples from physical education, such as being able to accurately measure, understand and use measurement data (e.g., heart rate change)
- reinforcing shape and position concepts in creating, understanding, describing and interpreting visual art, dance and music
- reinforcing use and interpretation of appropriate data display for category or measurement data within social studies, geography and history
- reinforcing number and data to critique claims and data displays, thinking about whose interests are served in how the numbers are presented
- link fraction knowledge to musical notation
- reinforcing transformation geometry ideas within teaching of art

(ii) Supporting teaching of material from mathematics and statistics within teaching and learning programmes of other disciplines

Aspects of mathematics and statistics are often used in the teaching and learning of other disciplines. However, the main focus of such teaching is the discipline other than mathematics and statistics, so the learning intentions, learning experiences and assessment are focussed on content of the other discipline. In our experience, opportunities for supporting learning of mathematics and statistics within other disciplines most often involve the application of selected mathematical and statistical knowledge, tools, and strategies from Number, Measurement and Statistics, as necessary for the learning material of the other discipline. Examples from Number include use of large and small real, fractional, and decimal numbers, proportions, percentages, operations on these, and rates. Examples from Measurement include making and reporting measurements, using scales, considering accuracy and making estimates of measures. Examples from statistics include sorting, analysing, and displaying data, although often with graphs and values that are different from those in the current statistics curriculum. In such teaching, teachers largely make use of the understanding students bring, rather than teach these areas of mathematics and statistics within their teaching of other discipline/s. In such instances, transfer of learning from one context to another is often a problem, in that ākonga can do something ‘in maths time’ that they can’t or won’t draw on in ‘science time’. This issue can be deliberately addressed in both contexts by teaching for using these links, including through context-based teaching, while keeping the integrity of the mathematics and statistics.

While mathematics and statistics knowledge and skills are likely to be supported when they are used in the service of other disciplines, the understanding necessary for deep learning, retention and understanding of the important connected nature across mathematical and statistical ideas are unlikely to be able to be developed well for students when the focus of the work is on another discipline. Again, making such links with effectiveness and integrity in teaching and learning programmes requires sound content and pedagogical content knowledge of the discipline being taught and mathematics and statistics.

(iii) Teaching mathematics and statistics within Integrated learning programmes

There is already much written about integrated learning and the affordances and challenges of this teaching approach.

Whichever approach is used (i-iii), maintenance of the integrity of each discipline in terms of curriculum emphasis, content and pedagogy is essential. For example, primary teachers are well versed in the content and emphases of the various curriculum areas and are well placed to make such links in their teaching in ways that support the learning of the other disciplines, and that support the learners they are working with. In our experience, many secondary teachers are less aware of curriculum-specific emphases outside of their specialism/s. There is a danger that cross-discipline links may do more harm than good if they lead to confusion and undermining of students' understanding and confidence in what they have learnt.

Several further considerations are also important when considering cross-discipline links in relation to mathematics and statistics. Most teachers were taught mathematics and statistics prior to the current mathematics and statistics emphases, and while primary teachers have had professional development and resources to assist them with these, secondary teachers of disciplines other than mathematics and statistics will not necessarily be familiar with current thinking about mathematics and statistics content and teaching and current digital technologies important for learning and doing mathematics and statistics. Secondly, consideration must be given to the cognitive load for students which is likely to be higher while trying to develop mathematics or statistics understanding at the same time as new learning within another discipline. Teaching from more than one discipline at a time is also more challenging for teachers, given the deeper calls on pedagogical content knowledge required for teaching from more than one discipline. In compiling this report, our reflection is that greater 'within' subject integration is possible and necessary for enhancing mathematics and statistics teaching and learning. 'Intra' subject links, rather than 'inter' subject links are even more vital now that research and curriculum are recognising greater links between number, statistical thinking, patterning, and spatial thinking.

Recommendations:
In summary, our response regarding important cross-disciplinary links between mathematics and statistics and other disciplines is that such links should only be used by teachers when the integrity of the curriculum emphasis and content of each discipline is maintained. For mathematics and statistics, this includes emphasis on important links within the discipline. Cross-discipline links could also be used to provide pleasure in learning, engagement, or motivation. However, inter-disciplinary links should be approached with caution, including with consideration for managing students' cognitive load and provision of additional planning time for teachers.

Pedagogy - Considering technology throughout all year levels and strands and in relation to rapid changes and growth in computer science/ICT

This report takes a broad view of technology to include invented tools that can be used to support the learning and doing of mathematics and statistics. With this definition of technology as invented tools, culturally-linked important ways of describing, measuring and representing our world(s), are examples of technologies invented and tested over time that can be explored and used (e.g., ways of finding locations and describing ’place’).

Technology is broader than digital technology, including physical materials, structured representations, and equipment. Physical materials such as blocks and tiles can serve multiple learning purposes – as models representing contexts or as elements for constructions. Technology includes measuring instruments including rulers, weighing scales and clocks (analogue and digital). Incorporating these technologies into mathematical learning leads to knowing how to use and ‘read’ the scales of these instruments and generating measurements that contribute to further mathematical activity/learning in various areas. Similarly, grid patterns, arrays and even a construction such as a ‘One Hundred Board’ is a technology - a stable representation that helps organise and present important mathematical entities for learners, such as the place value system. In each case, introduction and use of the tool is needed to enable effective use of the tool to explore and interrogate mathematical or statistics ideas.

The role of technology is to help learners ‘get into’ important mathematics and statistics and to create pathways that can help bypass tedious calculations that can disrupt more important thinking. The role of technology is not to mask or trivialise the powerful mathematics and statistics being used or to ‘get out of doing’ mathematics and statistics. The important role of technology is in re-presenting information, often visual, in a new way, or organising and displaying information for further learning (examples are patterns and structure of mathematical entities, and statistical displays etc.). For example, simple calculators can act as a ‘cognitive friend’ for younger children by being a number generation machine that offloads some of the ‘memory’ effort and cognitive load. At higher levels of the school, technology such as spreadsheets and graphing technologies are digital tools for organising and manipulating large data sets, generating more complex models and providing a range of interfaces for students to engage with. Suitable technological tools are an important part of mathematics and statistics learning throughout Years 0-13. This view is implied in the Year level templates. As we are aware there is some variation in practice and philosophy, in some of the templates for higher year levels, we have explicitly indicated examples of where we feel some content should be explored with digital technology or with and without digital technology.

Recommendation:

- We strongly advocate for ready and equitable access to and use of powerful technologies throughout Years 0-13 suitable for exploring and using the mathematics and statistics included at each level.
- That the refreshed curriculum be explicit about the benefits of young children engaging with physical models of mathematical and statistical ideas as well as using digital tools.

Computational thinking - Aspects of computational thinking are present, but in the background, in the current learning area of mathematics and statistics. Examples of activities where this is evident but not identified as 'computational thinking' include; The Sieve of Erastothenes (Number strand), setting up a spreadsheet (Algebra strand for patterns, tables, and graphs), instructions for a journey/network map (location - Geometry), metaphor of a number machine (Number and Algebra), steps for combining transformations to generate complex shapes for further transformation or tessellation (Geometry), and in the senior secondary school, spreadsheets, graphing software/graphical calculator provide statistical modelling tools to use with complex data sets. An approach that brings forward aspects of computational thinking, without overlap with the *Digital Technologies curriculum*, will better focus on the process aspects related to modelling and solving complex problems. Flow charts are accessible visual tools for supporting this process. Computational thinking involves processes of decomposition - breaking a complex problem down into parts, finding a pattern that shows a logical sequence of steps to be followed and possibly repeated, and testing and refining the process (sometimes producing an algorithm). This approach to modelling situations within existing mathematical and statistical content is a possible way forward for including a focus on computational thinking within the curriculum refresh.

Pedagogy - Further thoughts on pedagogy suitable for teaching and learning mathematics and statistics

This report has been prepared at time when there have been strong but mixed messages regarding suitable teaching methods for mathematics and statistics and therefore some confusion amongst teachers, families and the wider public. In addition, since the 2007 curriculum further tools and expectations of teachers regarding pedagogy and interactions with ākonga and families, such as *Tātaiako* and *Tapasā*, have been provided to teachers, and further literature on culturally sustaining, responsive and relevant pedagogy and assessment is now available. Ākonga experiences of learning the 'what by when' of the curriculum are heavily influenced by the pedagogies kaiako use. While consideration of pedagogy is largely outside the brief for this report, thinking about suitable pedagogy for enhancing access to learning and reducing barriers to learning has influenced our thinking and responses. Hence, we have included some emphasis consistent with suitable and equitable pedagogies throughout this report. To be clear, there is no one way to effectively teach mathematics and statistic. Students are best served when they are able to explore mathematical and statistical ideas in many diverse individual, pair-based and collective ways. They are underserved in learning situations in which only a limited range of pedagogies are present.

Recommendations:

- We strongly advocate for a clear national statement about the use of pedagogies suitable for teaching ākonga mathematics and statistics, consistent with *Tātaiako* and *Tapasā* and current thinking about culturally sustaining pedagogy.
- We strongly advocate for further information and guidance for teachers regarding effective pedagogies for teaching and learning of mathematics and statistics to be included in the curriculum writers' brief and work.

Final comments

This report addresses the Ministry of Education's request for advice on the **mathematics skills and knowledge learners need to know and by when, important cross-disciplinary links, and considerations in light of rapid changes and growth in computer science/ICT.** In doing so it also goes beyond this brief by surrounding this information with a number of key considerations and additional information. This is because the task of producing a 'what by when' list is somewhat fraught: such a list can easily become a checklist of skills taught in isolation, the foundation of high stakes testing or reporting or a proxy for the curriculum. In practice the key skills and concepts will not emerge unless they are taught in a rich and broad mathematics and statistics learning experience because many of them are about joining ideas together, recognising patterns, similarities, and the power of mathematical structures. For these reasons, users of this document should guard against extracting a shortlist of our signpost suggestions. We ask readers to consider this work in relation to the metaphor of a landscape introduced earlier with our suggestions as signposts.

We repeat here the caveats on this report arising from the way it was derived. In particular, this document does not represent a bicultural view of these issues. Notions of progress, development and cross-disciplinarity have cultural underpinnings and these have not been discussed or challenged from a Māori perspective, nor is the template framework used bicultural at its foundations. The Ministry of Education needs to remember this when it uses the information in this report. We do not want our efforts to undermine a genuinely bicultural development process for the curriculum, which we completely support. Our report draws from and is consistent with themes, findings, and key ideas from mathematics and statistics education research and literature. However, the time given for preparation of this report has not enabled us to do justice to suitably showing how selections of literature have informed this writing. In addition, consultation on our suggestions with an appropriate reference group has not been possible. We have not been able to consult with a range of others to get diverse views on our suggestions. Appendices Three and Four contain commentary on statistics and senior mathematics from some key thinkers in these areas. These reflections contain useful information for the next step in the curriculum writing process and are therefore shared in full in the appendices.

The Ministry of Education has the difficult task of drawing together a range of advice from different sources to refresh the mathematics and statistics curriculum. We have tried to dovetail our suggestions with some of the sources we had access to. Coherence across advice to teachers and between curriculum and assessment is going to be critical to ākonga success as a result of the curriculum refresh. One difficulty posed by the mathematics and statistics curriculum area is that there is clear evidence that two-year long levels is contributing to slow progress for our ākonga. Moving to longer bands of time will exacerbate this. A means needs to be found for mathematics and statistics to specify goals and minimum outcomes for each year of schooling (and after two terms at school) within the need for consistency of presentation across curriculum areas.

Finally, this report represents our best thinking within the timeframe allowed. It should be seen as a starting point for further work and discussion rather than a complete and flawless 'master list' of ideas.

References

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Appendix One: Continuity Overview Tables

These tables provide information extracted from the templates in a year-by-year format to make the focus suggestions easier to see as a progression. The statements are too broad to be used as signposts because each of them implies a number of ideas and skills that ākonga need. These are umbrella expressions of progression to give context to the signposts.

	Year's ākonga motto
Two terms	**I bring useful knowledge and understanding to my mathematics and statistics learning.**
Year 1	**In mathematics and statistics I am looking for patterns and the way things are related to each other.**
Year 2	**I can use pattern, number sense and simple statistical investigations to solve problems and learn new ideas.**
Year 3	**I understand how the place value structure of the number system works and can use this in exploring measurement and statistical situations.**
Year 4	**I understand that there are patterns in operations on numbers and data displays.**
Year 5	**I am shifting to a multiplication-based understanding of how numbers work and use this to understand and express probability.**
Year 6	**I am relating multiplication to place value, rational numbers, measurement systems, number patterns, geometrical patterns and relationships, data displays, probabilities, and different types of problem situations.**
Year 7	**I can use my multiplicative reasoning, pattern recognition and number sense to solve problems, and to investigate statistical contexts and relationships, and critique solutions and conclusions.**
Year 8	**I can reason proportionally using relationships and properties of number, shape and data.**
Year 9	**I can use strong content knowledge to select appropriate solution processes in a range of mathematics and statistics contexts.**
Year 10	**I can use proportional reasoning in number, algebra, measurement, geometrical, and statistical contexts.**
Year 11	**I can see and use links between different areas of mathematics and statistics.**
Year 12	**I can use tools of mathematics and statistics to understanding and explore mathematical and real world problems.**
Year 13	**I can use a range of sophisticated mathematics and statistics strategies and tools for exploring important mathematical and real world problems.**

Number and measurement

	Main kaupapa/purpose/focus for ākonga
Two terms	refining and linking ideas of relative magnitude and subitising to the formal counting system
Year 1	understanding how amounts, numerals and number names are patterned and linked together, up to 100
Year 2	developing a sound understanding of relative magnitude of numbers and begin to join, separate, compare and group to solve problems using this understanding as base
Year 3	understanding the place value system, and relate to number and measurement contexts
Year 4	developing consistently reliable methods for addition and subtraction, and beginning to understand rational numbers (including decimals and percentages) (how they show parts of a number or region or amount)
Year 5	establishing a method for multiplication and using place value, fraction and decimal understanding in measurement
Year 6	using multiplicative approaches to understanding place value, rational numbers and solving problems
Year 7	solving problems using fractions, decimals, and percentages
Year 8	choosing appropriate calculations and number representations to solve whole number, rational number and integer problems
Year 9	developing confidence in their use of adding, subtracting, multiplying and dividing proportions, and in estimating and measuring
Year 10	being able to select from and use a range of number and measurement tools and strategies in number, measurement, algebraic, geometric and statistical contexts
Year 11	being able to confidently use all of the number tools and understandings needed for being a numerate citizen and for continued learning in mathematics and statistics

Algebra

	Main kaupapa/purpose/focus for ākonga
Two terms	making patterns, and seeing patterns in things, including shapes and colours
Year 1	recognising what 'the same' means with amounts and shapes and looking for patterns in what they are learning
Year 2	seeking and describing patterns in the number system
Year 3	knowing '=' means 'the same as' and being able to use it to show equalities
Year 4	developing a solid understanding of how addition and subtraction on whole numbers work - what they are asking you to do to numbers and what some of the underlying rules are about how they work
Year 5	using patterns to make predictions, and using inequalities to express relationships between numbers and expressions
Year 6	knowing how the four operations 'work' and beginning to use functional thinking to understand and describe two-variable patterns
Year 7	using tables, graphs and rules (words or numbers) to discover and describe sequential patterns
Year 8	representing and explaining mathematical relationships in different ways, including using letters to represent variables and unknowns
Year 9	making and solving algebraic equations and working with linear functions to solve practical problems
Year 10	understanding relationships between information in tables, equations, and graphs for simple patterns and functions
Year 11	using equations, rules and graphs to explore real situations, solve problems and make predictions

Geometry

	Main kaupapa/purpose/focus for ākonga
Two terms	making, seeing and talking about patterns in shapes
Year 1	being able to see and describe patterns that use simple symmetry and shapes
Year 2	recognise, describe, and draw different shapes, and being able to see and talk about the effects of turns (rotation) and flips (reflections) on shapes
Year 3	moving, combining and drawing patterns of combinations of 2D shapes, and extend to moving 3-D objects in space
Year 4	knowing the spatial features that shapes and objects have and using these to sort and recognise shapes and objects in different places and orientations
Year 5	describing and modelling equivalence and difference in the context of shapes as well as numbers
Year 6	using formal language of geometrical properties to describe shapes, transformations and spatial positions
Year 7	using specific geometrical language that enables precision in describing and replicating shapes and design
Year 8	understanding features of shapes and space and moving between different models/representations of 2 and 3D shapes and objects
Year 9	using technology and geometrical relationships to resolve real world problems involving 2D shapes, scale and symmetry
Year 10	using technology, including geometry software, and geometrical relationships to understand and resolve real world problems involving 2D and 3D shapes, objects and symmetry
Year 11	selecting and using geometrical tools to investigate geometrical situations involving shape, position and symmetry across measurement, geometry and algebraic contexts

Statistics and Probability

	Main kaupapa/purpose/focus for ākonga
Two terms	sorting things into categories that show something about the collection of things
Year 1	using comparison to discuss investigations and their outcomes, including the language of chance
Year 2	working with others to pose a question, collect category data, sort it, discuss what was found and understand that not all responses or outcomes were equally likely
Year 3	working with more than one variable that describes a data set that they have collected and making sense of others' descriptions
Year 4	working with more than one variable that describes a data set collected by others and exploring the likelihood of particular outcomes
Year 5	thinking critically about category data displays created by themselves and others
Year 6	collecting and using time series data as well as summary and comparison data, and being able to model the likelihood of outcomes in controlled probability situations (such as rolling two dice)
Year 7	working with relationships in and through data, describing the shape of the data, and using continuous numerical variables
Year 8	engaging in all phases of a simple statistical investigation involving comparison or relationship situations, using and critiquing their own data sets and data sets from others, and be able to explain why the results of a probability experiment might not be the same as a model would predict
Year 9	understanding that variation and chance affect results of statistical investigations and probabilities of events happening
Year 10	understanding how the design of an experiment, data collection, sampling and analysis impact on findings of an investigation
Year 11	being aware of and able to critique chance and data situations relevant to important issues in their everyday lives and make decisions based on this awareness

Appendix Two: Example showing possible Important Cross-disciplinary links for mathematics and statistics with Te reo Māori

It is timely to ensure all ākonga have a growing vocabulary of kupu Māori for mathematics/pāngarau and tauanga/statistics and have opportunities to learn and use these terms and sentence structures in English and te reo Māori. Such knowledge and use assists ākonga to see purpose for and relevance of mathematics and statistics in their everyday lives and enables consideration of differing worldviews. For te reo Māori learners it would be ideal to have new mathematics and statistics focus te reo Māori learning for each year. It is suitable for kaiako to have guidance and support for this aspect of the teaching and learning of mathematics and statistics, including to help broaden and deepen the inclusion of te reo Māori within mathematics and statistics learning programmes.

The kupu Māori below are suggested as discussion starters regarding what makes for suitable knowledge of mathematical terms in te reo Māori for all ākonga in Aotearoa New Zealand across year levels as part of the key curriculum mathematical skills and knowledge. That is to say, they are included in this report as provocations for consideration rather than as a definitive selection. Consideration has been given to the Languages curriculum focus areas, mathematics content at the year levels, and potential cultural priorities. Consultation with suitable experts is expected.

It is assumed that language knowledge such as this will be developed cumulatively by kaiako with ākonga as ākonga progress through year levels.

Year	**Focus areas of the Languages curriculum[26] (levels 1-8)**		
	Language knowledge Knowing ngā kupu Māori to be able to:	**Communication** Sentence structures including and building on:	**Cultural knowledge (historical and current)**
0	communicate using counting numbers up to 20	e.g., Use numbers in simple sentences How many things? There are three things. How many people? There are four people.	Considering culturally located ways of thinking about quantity (how many) and that these can vary with worldview.
1	describe relationships between people	e.g., Use relationship words in sentences (e.g., mother, father, sister, brother, koro, kuia, tupuna, friend...) Who is your mother? Tere is my mother. How many sisters do you have? Who are your sisters?	Considering the personal relationships that are important to ākonga and their families and whānau.
2	describe position and following	e.g., Use position words in sentences (e.g., above, below, behind, in front of, on top of, under, beside...)	Considering culturally located ways of thinking about position and

[26] https://nzcurriculum.tki.org.nz/The-New-Zealand-Curriculum/Learning-languages/Achievement-objectives#collapsible1

	instructions related to position	Where is your bag? It is behind the chair. Stand beside the door.	instructions/commands and that these can vary with worldview.
3	describe the measure of an object and relative measures of objects	e.g., Use measurement words in sentences (e.g., long, short, hot, cold, longer, shorter, hotter, old, heavy, heaviest, big, small, smallest…) This is a long pencil. My cup is hotter.	Considering culturally located ways of thinking about quantity (measure) and that these can vary with worldview.
4	describe and communicate about simple shapes and spatial features	e.g., Use words for shapes in simple sentences (e.g., koru, patiki, circle, square, triangle, rectangle straight, angle…) What shape is the sandwich? The sandwich is a rectangle. These shapes have curves. Those shapes have straight edges.	Considering how features of shapes make shapes useful for particular purposes and explore how everyday objects utilise these properties.
5	describe and communicate about probabilities of events occurring	e.g., Use probability words in sentences (e.g., certain, never, likely, unlikely, 50:50, almost certain, impossible…) What are the chances of going to the shop today? Very likely. It is unlikely to rain tomorrow.	Considering likelihoods of events within culturally linked activities, e.g., in kapa haka competitions, powhiri, Iron Māori
6	communicate using names of simple fractions	e.g., Use fractions in simple sentences (e.g., falf, quarter, three quarters, a third, two thirds) I ate half the chocolate. What is the time? It is quarter past ten.	Considering culturally located ways of thinking about sharing.
7	describe and use compass directions	e.g., Use direction words in simple sentences (e.g., north, south, east, west…)	
8	…		
9			
10			
11			
12			
13			

Appendix Three: Dr Pip Arnold's notes

What is in my document:

The indicators for statistical investigations and statistical literacy

These were developed a couple of years ago to try to unpack in much more detail the current achievement objectives. The statistical investigation ones have been constantly updated and tweaked as I am writing my book based on what I have found in my research (both PhD and reading extensively) for the different phases of the statistical enquiry cycle.

I know for statistical investigations the indicators have been pass many mathematics and statistics and pāngarau facilitators, and mathematics and statistics teachers. So, they have been checked in the sector. I just can't remember if I included statistical literacy or not in those sessions, I don't think so, but they were developed by Maxine and myself mostly, I may have also had input from others, suspect at least my MLC primary group of teachers will have looked at it.

I am using the statistical investigations indicators in my book to frame each part of the PPDAC cycle and then do some even deeper unpacking by curriculum level around specific concepts or parts of a phase of the cycle. E.g., investigative questions, data visualisations, reasoning about data, data ethics, what to measure and how. I also refer specifically to the level 5 statistical literacy indicators in the book to make the connection across the threads of the statistics strand.

How it might look at different year levels
For statistical investigations I have in the end probably done more of a brain dump as to write them coherently and follow a logical flow takes a long time and needs discussion with others, so the second column for statistical investigations is some initial thinking about how it might look at the different levels. Some of these I have already done work with teachers in schools, e.g., for year 7 & 8 I have worked with 9(2)(ba)(i) and shared ideas with year 7 & 8 teachers in my MLC group. For year 9 & 10 I have worked with Lynfield College and Northcote College to update what they did in years 9 & 10 to make it more meaningful and not just the same thing repeated.

The ideas for year 5 & 6 are based on some work I did with a year 5 teacher (for their year 5/6 syndicate and we very much pulled on the CensusAtSchool questionnaire as foundational to the purpose for the statistics investigations in that year, hence the suggestion to alternate between odd and even years for curriculum level 3 and between statistics and probability.

At level 1 and 2 I am not so knowledgeable having not yet been through the process with a school though that is something I will have done by early next year, currently working in a year 1-8 school to review their entire mathematics and statistics curriculum, so the opportunities will arise there. A few notes based on some initial reading for year 1, and as you know ECE -early primary is in our sights, mentioned in previous meeting with maths strategy group (NZSA ed com meeting).

Sometimes being able to make a split between two years in a curriculum level is easy as there is a wide choice of situations and data source/collection. This choice should be up to individual schools to make based on topics of interest that might form the contexts they use. For example, at 9(2)(ba)(i) they decided in year 7 to focus on travel and a social studies topic as the contexts for the statistics and sport for year 8. Another school with different contexts might explore the combination of situations differently.

I have tried to identify what are the ***big new things*** at each of the levels, it is clear what the progression steps are when you look closely.

Statistical investigations

Curriculum level - indicators	How it might look at different year levels
Curriculum level 1 Uses the statistical enquiry cycle to conduct investigations • with the teacher, areas of interest are explored • with the teacher, investigative questions about ***categorical data*** are developed • with the teacher, how they will gather, sort, and count the data are designed • with the teacher, collects data • creates visualisations to illustrate the data they have collected	At curriculum level 1 (year 1 & 2) students are introduced to simple ***summary situations*** using ***categorical variables***. The teacher is working closely with the students through all stages of the PPDAC cycle.
	Year 1 In year 1 students are learning about (1) a need for data, (2) the benefit of recording to remember and (3) a need to analyse the data to answer the investigative question (Makar, 2018[27]).

[27] Makar, K. (2018). Theorising links between context and structure to introduce powerful statistical ideas in the early years. In *Statistics in Early Childhood and Primary Education* (pp. 3-20). Springer, Singapore.

<table>
<tr>
<td rowspan="2">
◦ Visualisations could be individual case plots (e.g., name on the horizontal axis and number of children in family on vertical axis, rather than the number of children in family on horizontal axis and frequency on the vertical axis)
◦ Groupings of similar objects
• Makes statements about their displays
◦ Likely to be about individuals
◦ Need to reflect what the data is showing
• with the teacher, answers the investigative question
</td>
<td>Simple summary situations using categorical data are the focus and are only limited by the everyday experiences that students have. No fixed requirements for data visualisation.</td>
</tr>
<tr>
<td>Year 2

In year 2 students could be exploring the data collected by others as well as by themselves. Recognising that data exploration doesn’t have to be just data we have collected. This would entail them needing to understand what data was collected, and the purpose of someone else’s need for data.</td>
</tr>
</table>

<table>
<tr><th>Curriculum level - indicators</th><th>How it might look at different year levels</th></tr>
<tr><td rowspan="3">Curriculum level 2
Uses the statistical enquiry cycle to conduct investigations
with the teacher, areas of interest are explored
with the teacher, investigative questions about categorical and discrete numerical (whole number) data are developed
with the teacher, how they will collect the data is designed
collects data from the class or using secondary data sources to answer the investigative question
records data using a variety of methods, e.g., using data cards, tables, tally charts
Ideally for secondary data sources uses multivariate data cards to allow a variety of variables to be explored (3-5 variables, e.g., four on the card and then the colour of the card another categorical variable with only a few outcomes)
makes data visualisations where the variable of interest is on the horizontal axis and the data points (e.g., dots, cards, sticky notes) represent the frequency vertically (vertical axis if used)
makes summary statements about the data, connecting it to the group that was investigated
Reads the data, e.g., three students in our class have four children in their family
Reads between the data, e.g., five students in our class have one or two children in their family
Teachers encourage students to read beyond the data by asking questions such as: “If a new student joined our class, how many children do you think would be in their family?”
answers the investigative question</td><td>At curriculum level 2 (year 3 & 4) students are expanding the range of data that they collect to include discrete numerical variables (counts). Students are collecting data using different tools or working with existing multivariate datasets.</td></tr>
<tr><td>Year 3

Whereas in years 1 and 2 data exploration has probably centred on one variable at a time, both in the collection and analysis, in year 3 students are starting to explore data using multivariate datasets. Their investigation purposes are likely to require a need for data about more than one variable to get a wider picture of what is going on. They may investigate multiple variables in a statistical investigation.

In their analysis they are using data visualisations such as dot plots and bar graphs using provided grids or data cards. When describing the data, they are starting to combine responses e.g., the number of families that have three or more children, the number of students who travel to school by a motorised vehicle (e.g., car and bus).</td></tr>
<tr><td>Year 4

The main difference between year 3 and year 4 students will come down to the complexity of the context they are exploring, and a changed focus to working with multivariate secondary data, data collected by others.

They will be using similar tools to those used before, making decisions about which visualisation is best to use. Their analysis statements should be connecting to the group that was investigated, and they should be starting to think beyond the data in hand.</td></tr>
</table>

<table>
<tr><th>Curriculum level - indicators</th><th>How it might look at different year levels</th></tr>
<tr><td rowspan="2">Curriculum level 3
Uses the statistical enquiry cycle to conduct investigations

identifies broad area to explore using the statistical enquiry cycle
poses investigative questions about summary (categorical and discrete numerical) and time series situations
makes predictions/assertions about what they expect to find out
designs how they will collect the data (with teacher guidance)
collects data from the class or a wider group (several classes) - primary data
records data systematically
uses data collected by others - secondary data
secondary data sources that use multivariate data cards or datasets to allow a variety of variables to be explored
software such as CODAP is used to share multivariate datasets
using a variety of data displays e.g., dot plots, bar graphs
exploring a second variable, for example, by using colour
using statistical software
Uses a variety of data visualisations, e.g., dot plots, bar graphs, frequency tables, line graphs
Explores a second variable, for example, by using colour
Makes summary statements about the data, connecting it to the group that was investigated
Identifying patterns in context
Reads the data, e.g., the most common number of children in a family in our class is three, nine students in our class have three children in their family
Reads between the data, e.g., most students (16 out of the 27) in our class have between two and four children in their family
Identifying trends in context
</td>
<td>At curriculum level 3 (year 5 & 6) students continue to work with categorical and discrete numerical variables. The investigative situations are expanded to include time series and comparisons.

Time series situations should be straightforward, based on topics of interest and connect across the curriculum.
Visual comparison of groups for the same variable is possible but do be aware of the size of the groups as different size groups require proportional reasoning to compare, whereas groups of the same size can be compared by counts.
Using software to explore data should be introduced at this level if it hasn’t been introduced earlier. Students can explore data using the software tools and go through mini exploratory data cycles, e.g., wondering about a variable, making a data visualisation, wondering some more, making a new data visualisation – coming to an end point with a purpose for their investigation.

Across the two years of curriculum level 3 a choice to split the curriculum might be in the focus of the enquiry, one year it has statistical investigations as a focus (and this might align with CensusAtSchool data collection year – odd years); the other year probability investigations as a focus. The data collection is for different purposes, to answer an investigative question using data or to explore a probability situation using a probability experiment.</td></tr>
<tr><td>Year A (odd years)

Exploring summary and comparison situations, deep dive into collecting data for many survey questions, using questionnaires, making measures, exploring data from CensusAtSchool.</td></tr>
</table>

▪ Reads the data, e.g., the height of the seedling after 10 days is 5 cm; the number of ice creams sold during summer is 1053 ▪ Reads between the data, e.g., there is a repeating pattern with more ice creams sold in summer and less in winter; the seedling appears to be growing quickly initially and after 10 days it is growing more slowly • Teachers encourage students to read beyond the data by answering questions such as: “If a new student joined our class, how many children do you think would be in their family?”; “We missed measuring the height of the seedling over the weekend, what do you think the measurements would have been for Saturday and Sunday?” • answers the investigative question	Year B (even years) Exploring time series situations – keeping the statistical enquiry process to the forefront, learning about simple probability experiments.

<table>
<tr><th>Curriculum level - indicators</th><th>How it might look at different year levels</th></tr>
<tr><td rowspan="2">Curriculum level 4
At this level learning experiences should include students using the statistical enquiry cycle to conduct investigations by:
• identifies broad area to explore using the statistical enquiry cycle
• poses investigative questions about summary, comparison, relationship, and time series situations
  ○ makes predictions/assertions about what they expect to find out
• plans for data collection using surveys
  ○ determines variables needed to answer investigative questions
  ○ plans how to collect data for each variable
    ▪ poses data collection questions
    ▪ decides how to make accurate measures when needed
  ○ collects data from the class or a wider group (where all the wider group can be surveyed)
• sources and uses data collected by others - secondary data
  ○ interrogates the data e.g., what were the survey questions posed? Who was the data collected from? How was the data collected? What is the variable? How was it measured?
• explores summary, comparative, bivariate and time series situations
  ○ uses multiple representations to analyse and visualise data
  ○ using statistical software to analyse and visualise data
  ○ explores numerical and categorical data
  ○ describes distributions (summary and comparison - categorical and numerical), relationships (bivariate – paired categorical and paired numerical), and trends (time series)
• communicates findings using the entire statistical enquiry cycle
  ○ answers the investigative question using evidence from their analysis</td>
<td>At curriculum level 4 (year 7 & 8) students are working with all the investigative situations: summary, comparison, time series and relationship. Continuous numerical variables are introduced at curriculum level 4.
• Relationship situations are new, and it is recommended that students are introduced to relationship situations for paired numerical variables where the relationship is represented by the y=x line, for example, height and arm span.
• For summary situations students can be introduced to the idea distributional shape for numerical variables through large datasets. Visually locating the middle of the data and circling the middle group of the data are both foundational steps towards the formal introduction of box plots at curriculum level 5.
• Students should still be exploring summary situations for categorical data, with a focus on reading between and beyond the data.
• For comparison situations students are comparing visually, using the tools they have developed for summary situations. They are comparing features of the two distributions.
• For time series situations students are starting to explore related time series, for example, monthly average temperatures between two locations, in different parts of NZ or different parts of the world.
Across the two years of curriculum level 4 the situations can be split e.g., summary and time series in year 7, comparison and relationship in year 8. There can be more of a focus on collecting primary data in one year, while the other year can use more secondary data. A suggested split is given.</td></tr>
<tr><td>Year 7

In year 7 students are independently using the PPDAC cycle to explore and investigate summary and time series situations.</td></tr>
</table>

	They are ***planning for and collecting data*** about their class or another defined group that they have access to. They are learning about how to determine variables to answer their investigative question. Students are learning how to use questionnaire tools such as Google Forms to collect data. Students in year 7 are using sourced secondary data to create data visualisations, and using existing data visualisations e.g., from FigureNZ, StatsNZ | Tatauranga Aotearoa, Gapminder. Students should be using statistical software such as CODAP[28] to make data visualisations. Data visualisations include ***dot plots, bar graphs, hat plots, time series graphs,*** frequency tables.
	Year 8 In year 8 students are independently using the PPDAC cycle to explore and investigate ***comparison and relationship situations***. A bigger focus on using ***sourced secondary datasets***, how to interrogate the data collected by others, identifying the variables available and posing investigative questions that they can answer with the dataset provided. Data visualisations include frequency tables; ***side-by-side*** dot plots, bar graphs, and hat plots; and ***scatter plots***.

[28] Common Online Data Analysis Platform – free web based statistical software that allows a lot more flexibility for students to play with and explore data https://codap.concord.org/

<table>
<tr><td>Curriculum level - indicators</td><td>How it might look at different year levels</td></tr>
<tr><td rowspan="2">Curriculum level 5
Uses the statistical enquiry cycle to plan and conduct investigations

poses investigative questions about populations for summary and comparison situations
makes predictions/assertions about what they expect to find out
poses investigative questions for relationship and time series situations
makes predictions/assertions about what they expect to find out
plans for data collection using surveys

determines the variables needed to answer investigative questions
considers sources of variation
decides how to measure variables
considers ethics
poses data collection and survey questions

undertakes data collection using surveys

collects data from a sample
takes samples that are likely to be convenience samples e.g., class; or ones that are generated using random sampling e.g., CensusAtSchool samples; sampling design not expected at this level
sorts, cleans and recategorises data

sources and uses data collected by others

interrogates the data e.g. What were the survey questions posed? Who was the data collected from? How was the data collected? What is the variable? How was it measured?
sorts, cleans and recategorises data

Explores summary, comparative, relationship and time series situations

Uses multiple representations to analyse and visualise data
Explores numerical and categorical data
Recategorises data as needed to answer the investigative question

</td>
<td>At curriculum level 5 (year 9 & 10) students are expanding the range of statistical tools that they have to describe the features of the different situations. Sample to population inference for summary and comparison situations (only) is new at this level.

Comparison situations – learning about how to make an informal statistical inference about what is happening back in the populations based on samples. Learning about sampling and early notions of sampling variability.
Relationship situations – learning about how to decide if there is a positive or negative association (or no association) using tools such as the quadrant count ratio (QCR), identifying the trend of the data
Time series – introducing time series that have seasonal variation
Experiments are introduced.

As with previous levels what is covered within a year program can vary between the two years. A suggested approach is given.</td></tr>
<tr><td>Year 9

Using summary situations to introduce the need for taking a sample from a population. Exploring ideas about sample size needed for categorical variables versus sample size needed for numerical variables. Working with distributions and learning about and identifying the features that can be described. Learning about measures of centre and measures of variation as they apply to summary situations – for categorical and numerical variables. Learns about recategorising data to better answer the investigative question or to explore a new investigative question.

Exploring relationship situations using tools such as QCR and learning about and identifying features that can be described for relationship situations.</td></tr>
</table>

- o Describes distributions of samples (summary and comparison), relationships of group (relationships - paired categorical and paired numerical) and trends (time series) - o Uses measures of centre (e.g., median), spread (e.g., IQR, interquartile range), proportion (e.g., proportion of girls who walk to school) - o Find the QCR, quadrant count ratio (relationship) - o Uses visual evidence to communicate features in context - o "makes the call" when using samples to answer investigative questions about populations - o answers the investigative question using evidence - presents a report of findings using the whole statistical enquiry cycle - o integrates the statistical and contextual - o provides explanation or interpretation - o justifies their findings Uses the statistical enquiry cycle to conduct experiments - poses investigative questions that can be answered using experiments - plans the experiment - o determines control and response variables - o designs experiments to collect data to investigate the situation - undertakes the experiment - uses multiple representations to display results of experiments - presents a report of their findings using the whole statistical enquiry cycle	Focus on collecting data especially on posing good ***data collection/survey questions**,* considering how to "measure" different variables, what instructions are needed, what survey question to ask. Descriptions are fully connected to the context of the investigative question. Students are learning to connect with the context through explanation and interpretation. **Year 10** ***Comparison situations**,* learning about *"**making the call**"* about what is happening back in the population. ***Box plots*** are introduced and associated features that can be used to describe the distribution. Recognising the need for multiple visualisations to show different features of the distributions. Sourcing secondary data to use allows the focus to be more on the analysis and inference, while allowing students to still think about the important aspects of planning through interrogating the dataset. Simple ***experiments*** are introduced and ***time series*** with seasonal variation are explored.

Statistical literacy

CL1

At this level learning experiences should include students interpreting statements made by others from statistical investigations and probability activities by:

- listening to others as they describe what they see in simple data displays
- identifying the relevant features of the display that others are describing
- improving on descriptions of others if they can

CL2

At this level learning experiences should include students comparing statements with the features of displays of statistical investigations and probability activities undertaken by others by:

- looking at the display (tables, graphs) and reading the statement about the display and deciding if the two are in agreement
- if in agreement connect the display feature to the statement
- if not in agreement identify where the statement doesn't align with the display and make a suggested correction

CL3

At this level learning experiences should include students evaluating effectiveness of displays in representing findings of statistical investigations and probability activities undertaken by others by:

- interrogating the displays using prompts such as
    - What types of displays were used to analyse the data? (Tables, graphs, calculations, probabilities, hand-drawn, software generated)
    - Can you read the display?
    - What do the displays tell you?

- Do you understand what the display is telling you about the statistical investigation or probability activity?
- Were the displays chosen the best ones to tell the story about the statistical investigation or probability activity?
- What displays would you do instead?
- What displays would you do as well?
- Were the displays misleading in any way?

CL4

At this level learning experiences should include students evaluating statements made by others about the findings of statistical investigations and probability activities by:

- interrogating statements made by others in their analysis and conclusions using prompts such as:
    - ANALYSIS
        - Do the statements about the displays accurately reflect what the displays are showing?
        - Do the statements about statistics accurately reflect any statistics given?
        - Do the statements contain contextual information e.g., variable, values, units, group?
        - Is there another analysis that could have been done?
            - Could other variables have been considered?
            - Are there different displays that could have been drawn?
    - CONCLUSION AND COMMUNICATING FINDINGS
        - Has the investigative question been answered?
        - Have ideas and findings been communicated clearly?
        - Do you believe the findings? Why? Or why not?
        - Do the findings raise further questions?
- suggesting improvements to others analysis and conclusions

- interrogating statements made by others about probability activities[29]

CL5

At this level learning experiences should include students evaluating statistical investigations and probability activities undertaken by others by:
- evaluating the work of others by looking at all the phases of the PPDAC cycle for statistical investigations
    - PROBLEM
        - What is the investigative question?
        - Is the investigative question about a group (all the data will be used) or about a population (a sample will be used)?
        - What is the group or population of interest?
        - Did any predictions made align with the investigative question?
        - Is it clear what sort of situation it is from the investigative question (summary, comparison, relationship, time series)?
    - PLAN
        - Did they collect their own data, or use secondary data?
        - Did they collect a representative (random) sample?
        - How did they collect the data and where and when did they collect it?
        - Who collected the data?
        - What survey questions were asked (own data) to collect the data?
        - Were the survey questions appropriate? Did they provide data to answer the investigative question?
        - What were the variables of interest?
        - How were the variables of interest measured?
        - How were the measures defined?

[29] This work not done yet in detail, was waiting to see what happened at the top end with probability before reshaping lower down.

    - What was the sample size?
- DATA
    - How was the data collected and recorded?
    - If secondary data is used, what is the source of the data?
    - What type of data is it, categorical or numerical?
    - Did the data require cleaning?
- ANALYSIS
    - What displays were used to analyse the data?
    - Were the displays chosen appropriate for the data and the investigative question?
    - Were the displays misleading in any way?
    - What statistics were given?
    - Do the statements about the displays accurately reflect what the displays are showing?
    - Do the statements about statistics accurately reflect the statistics given?
    - Do the statements contain contextual information e.g., variable, values, units, group or population?
    - Is there another analysis that could have been done?
- CONCLUSION AND COMMUNICATING FINDINGS
    - Has the investigative question been answered?
    - Was the sample size big enough to make such a conclusion (understanding the difference between sample size needed for numerical data and sample size needed for categorical data and being)
    - Do the conclusions make sense considering the analysis?
    - Has reliable evidence been used to make their conclusions?
    - Have ideas and findings been communicated clearly?
    - Could there be alternative explanations for the findings?

    - Do the findings raise further questions?
- evaluating the work of others by looking at all the stages of probability activities

Appendix Four: Anna Fergusson's quick feedback

Overall

I don't think PPDAC should be used anywhere in the documents at all levels. The descriptions should refer to the statistical inquiry cycle instead if needed, otherwise just describe the statistical investigations.

Overarching overall statement ideas for each year level would be based on developing more sophisticated reasonings with data and models (including probability models), moving from awareness and participation to production and creation, all the time expanding their toolkit for critically evaluating arguments based on data, studies, visualisations.

I don't think the purposes of investigations should be labelled as "important" or "societal issues" because this would limit fun, creative investigations that are personally or culturally relevant for students. I also don't think this captures the "step up" from Year 11 to Year 13 (see earlier comments)

What might be missing from the current curriculum? What would be your main points that you would like to see included?

- The current AOs were written back in 2006 and the world of data and technology as changed so much since then and will continue to do so. Our ākonga need a wider variety of learning experiences with data to equip them for the modern world. We need to anticipate future changes and try to pull out the key ideas that will remain as best we can in an ever-changing world.
- The current AOs do not clearly describe the *different reasons* for obtaining and analysing data e.g., for description, estimation, prediction, explanation and control/intervention. These overarching purposes will be developed across all curriculum levels. Sitting under these are different types of variables, displays and ways to analyse data, including using models
- Statistical investigations a key aspect but these do not all follow the same PPDAC format at the higher levels (and I would also suggest the lower levels). For example, modelling-focused investigations are different, learning-about-modelling investigations are different, similarly data science and probability modelling investigations are different. See previous bullet point.
- Probability is a key area for rewriting/refreshing. At CL7 and CL8, we have moved (in line with statistical practice) to a modelling perspective. This requires understanding that the true probability may not be the same as the model probability, and that data that is collected from the situation being modelled is not the same as data generated from a probability model. Unfortunately, traditional notions of "theoretical" and "experimental" probability are still retained in the curriculum documents at the lower levels and there is still confusion. We also need to promote more authentic probability modelling experiences.
- Statistical literacy now has to includes data literacy, and needs to extend beyond traditional reports to any form of communication, including visualisations, videos, news shows, etc. We also need to ensure that ākonga are familiar with algorithmic models and processes, how data is used to develop these, and issues with algorithmic bias. Essentially, that humans play an important role in data-driven decisions, in both positive and negative ways.
- We don't devote enough time and space to develop data exploration and visualisation skills at the senior levels. Data journalism, data science, data visualisations, you only have to look at all the communications related to COVID to see how important these skills are. We need space in the curriculum for ākonga to create and be creative, not just follow rules and meet criteria for writing things a certain way.
- We don't currently include any modern data and data science related methods, such as machine learning (supervised or unsupervised), visual modelling approaches such as networks and interactive documents, integrating statistical and computational thinking including different tools for exploration, visualisation and modelling.
- We have a strong focus on traditional statistical inference, which is still important, but has been limited to specific types (e.g. difference between two medians), rather than building general ideas around concepts like: using intervals to express uncertainty based on sources of variation like sampling (e.g. confidence

intervals, predictions intervals, comparison intervals), using tests based on "just chance explanations" in order to make calls or investigate claims (e.g. CL5/6 guidelines, randomisations tests)

- Designing and using experiments remains an important idea to focus on at CL7 and CL8.
- Other aspects like modelling relationships between variables, including time series and linear models, can be developed across CL6 to CL8 – currently they do not feature at CL7.
- Some current features of the AOs at CL6 to CL8 are artefacts from previous versions and need re-evaluating, for example, why does questionnaire design only feature at CL7? Why is just this form of collecting data privileged above other forms of collecting data? It is an important tool but using questions with humans to collect data happens across all year levels!
- The role of technology is only hinted at currently in CL8, with reference to resampling and randomisation techniques, although these can be done without technology. We need to ensure that data science (e.g. integrating statistical and computational thinking to learn from modern data) is supported and encouraged by the refreshed curriculum, and this will require careful specification of what ākonga need to know and how to learn this knowledge. There should not be a reliance on other learning areas for technology learning, e.g. digital technologies, as we require a maths/stats specific articulation of what it means to develop models from data using statistical, computational and mathematical thinking.

Year 11 to Year 13 table – suggested descriptions...

Year 11	Year 12	Year 13
Statistics investigation • Use statistical investigations to answer investigative questions, **justifying variables and measures used in the data collection phase taking variation and uncertainty into account, and integrating statistical and contextual knowledge (all levels surely?)**, • Use informal methods to make inferences about populations, using features of sample distributions • Explore data using multiple displays, discussing features of the distributions (e.g., trends, relationships between variables, differences within and between distributions), and making connections to statistical and contextual information	Statistical investigation • Use statistical investigations to answer investigative questions, using data from existing digital sources, surveys and experiments, discussing implications of findings and potential recommendations for action • Use informal methods to make inferences about population parameters, using comparison intervals • *Why isn't exploratory data analysis discussed? It is a current AO*	Statistical investigation • Use statistical investigations to answer investigative questions, using data from existing digital sources, surveys, experiments, and observational studies, seeking explanations and evaluating all stages of the cycle, and discussing implications and recommendations • Use statistical models (e.g., including linear regression for bivariate data and additive models for time-series data) to make predictions • Use formal methods to make inferences from surveys and experiments by determining estimates and confidence intervals for means, proportions, and differences, and using methods such as randomisation to assess the strength of evidence *Why isn't exploratory data analysis discussed? It is a current AO*
Statistical literacy • Interpret and evaluate *everyday* statistically based communications, using relevant knowledge from this curriculum level • ***NEW idea this level?*** Identify who the claims are being made about and whether the data	Statistical literacy • Interpret and evaluate *formal(?)* statistically based communications, using relevant knowledge from this curriculum level • ***NEW idea this level*** Critique risk and relative risk statements and use a variety of strategies to	Statistical literacy • Evaluate a *wide range* of statistically based communications, using relevant knowledge from this curriculum level • ***NEW idea this level*** Critique causal-relationships claims and

collected/presented supports these claims	communicate about risk (including visualisations)	interpret margins of error associated with polls/surveys
Probability • Explore chance situations and calculate probabilities in discrete situations, including whether theoretical chance outcomes are equally likely **(this doesn't seem enough)** • Investigate chance situations involving discrete random variables through probability experiments (e.g. recording results, plotting frequencies of outcomes and describing features of probability distributions)	Probability • Explore chance situations using representations such as probability tree diagrams and two-way tables to calculate probabilities • Model chance situations involving continuous variables, using probability distributions including the uniform and normal distributions • Use simulations to estimate model probabilities, supported by technology	Probability • Explore chance situations using probability concepts such as randomness, mutually exclusive events, and independence to calculate probabilities of combined and conditional events • Model chance situations using discrete and continuous probability distributions including the Poisson, binomial, triangular and normal distributions • Use features of discrete random variables, such as expected value and standard deviation

Annex 3: Questions and Answers for Ministry of Education

Why was the paper commissioned?

The New Zealand Curriculum is being refreshed by the Ministry of Education to make sure it is bicultural, inclusive, clear and easy to use – so that every ākonga can learn, develop and make progress in the things that really matter to them now and for their future. The refresh is being phased over five years, to help to make it more manageable for schools to implement the refreshed curriculum. Schools, ākonga and whānau will be supported through the change. The Mathematics and Statistics learning areas will be refreshed in 2022 along with English and Science and will be ready for use in 2023.

Recent data from national and international studies suggests that a significant proportion of learners in Aotearoa New Zealand are not experiencing success in their learning in Mathematics. The Ministry is taking a systems approach from Early Learning through to Year 13 that is aimed to result in long-term sustained growth in achievement in Mathematics for students in both Māori Medium and English Medium contexts.

The Ministry has done considerable work on the refresh process including wide-ranging engagement, and also requested the Royal Society Te Apārangi to establish a Mathematics Expert Advisory Panel to provide independent advice to progress the work,.

What was the panel asked to provide?

The Royal Society Te Apārangi Mathematics Expert Panel was commissioned to provide independent advice on the development of mathematics knowledge and skills in relation to *The New Zealand Curriculum*, specifically:

- the mathematical skills and knowledge needed for being a critically engaged citizen,
- the mathematics skills and knowledge learners need to know and by when, and the important cross-disciplinary links, taking into consideration the rapid changes and growth in computer science/ ICT,
- the important 'big ideas' in mathematics and statistics that all learners need to develop through schooling,
- the relationship between numeracy and mathematics, and
- progress, assessment and where the checkpoints in the mathematics education pathway should be.

How were the members of the panel selected?

The Royal Society Te Apārangi selected the members of the panel. They were selected to ensure a range of academic expertise in mathematics and statistics as well as mathematics and statistics education to allow them to provide the advice needed to inform the refresh of the Mathematics and Statistics learning area.

What advice and recommendations are included in the paper?

The final report offers insights into recent evidence about effective mathematics teaching and curriculum design. It describes the Panel's views on the key connections and important interdependencies between the intended and the experienced curriculum. The report is available on the Royal Society Te Apārangi website. It is comprehensive, offering advice and 14 recommendations for the Ministry to consider. The recommendations are presented in four broad themes:

- **Slippage**: this describes how year-on-year many ākonga fall away from the trajectory described in the Mathematics and Statistics learning area of the *NZC.*

- **Teacher discipline and pedagogical knowledge**: this refers to teacher knowledge of mathematics and statistics and how to teach this effectively.

- **Leadership**: this theme recognises the need for a more centralised approach to support coherence and clarity around teaching and learning in mathematics and statistics.

- **Inequity**: The evidence is that many groups of learners in our education system continue to experience inequitable outcomes. Achievement data in mathematics and statistics reflects this.

How will the Ministry use the advice?

Minister Hipkins first signalled the need for a stronger focus on progress in 2017, and since then the Ministry of Education has been working with people from the education sector and wider communities to understand how to make improvements to the national curriculum that will ensure our students succeed. The Ministry will draw on this knowledge as well as other expertise to co-design curriculum content at all stages of the refresh and the wider curriculum work programme.

The advice provided by the Royal Society Te Apārangi Mathematics Expert Panel along with information from a wide range of sector, community and expert representatives will help inform the refresh of the Mathematics and Statistics learning area and the Mathematics Strategy. The refresh of the Mathematics and Statistics learning area will follow a process of co-design. There is more information on the Ministry's website at: https://www.education.govt.nz/our-work/changes-in-education/curriculum-and-assessment-changes/new-zealand-curriculum/

What work is the Ministry doing to strengthen Mathematics teaching and learning?

As part of the refresh of *The New Zealand Curriculum* the Ministry has been working intensively on its strategy for teaching and learning mathematics and statistics (including numeracy). This year their teams have drawn on the views of Māori, Pasifika communities, employer and industry representatives, academics and experts, professional learning and development providers, members of the disability sector and English language learners. Peak Bodies of the schooling sector were asked for comment and advice was sought from the *NZC* Curriculum Voices Group and the Early Learning practitioner sector. Previous work done by the Curriculum, Progress and Achievement (CPA) Ministerial Advisory Group, Early Learning Ministerial Advisory Group and Education Conversation | Kōrero Mātauranga was also used to inform this part of the *NZC* refresh.

Ministry staff engaged with groups throughout the country during the year – leaders, teachers and kaiako, ākonga, parents and whānau, and the wider community. Further invitation for feedback and comment was also available through the Ministry of Education website.

They looked at "best practice" education initiatives and approaches from other countries to ensure as much relevant information as possible was used to inform the direction of the strategy.

I am pleased to note that some of the advice provided in this report reinforces the issues the Ministry had already identified as being critical. It aligns well with the thinking they've been doing and that has been endorsed and agreed through consultation with the sector and key stakeholders.

Have you done the same for other aspects of the NZC refresh?

In February 2020 at the request of the Ministry of Education, Royal Society Te Apārangi established an Aotearoa New Zealand's Histories Expert Advisory Panel to provide independent critique of new content developed for *The New Zealand Curriculum* and *Te Marautanga o Aotearoa.* This process provided rigor and value to the work programme.

How much did it cost taxpayers?

The Ministry paid reasonable and actual costs for panel members, including travelling and other expenses relating to the performance of the duties and responsibilities as members.

Annex 4: Questions and Answers for the Minister

Measuring achievement and progress and use of assessment tools, and the capability of the workforce.

1. How will the Ministry use this report to focus on achievement and progress?

The Ministry will use this report as part of a wider mathematics strategy and to inform the NZC refresh of the mathematics and statistics learning area.

2. How will updating the NZC help?

A learning progression framework of 'progress outcomes' which describe the learning that cannot be left to chance at each phase of a learning progression. This enables a view of the continuity of learning and progress over time, making explicit the increasing breadth and depth of learning.

3. How will we know that ākonga are making progress?

In 2022, NMSSA will provide us with information about the achievement of Years 4 and 8, and the difference between these two years. Other year levels may also be included in the 2022 study. This monitoring will be repeated soon after the mathematics and statistics learning area has been refreshed so we can see whether the changes in the system have supported teachers and learners. International studies such as TIMMS also provide us with information about student achievement.

4. How will the Ministry support ākonga who are not progressing with their peers?

There is currently support for teachers to accelerate learner progress including mentoring support. The Ministry is considering how it can strengthen this support, so it reaches more students.

5. What other support will teachers get to respond to ākonga who need extra support?

The nzmaths website is being updated with resources that are directly targeted for students, and others that teachers can pick up and use in the classroom.

6. Will there be new tools developed to assess maths learning?

The current tools are being audited to check they are fit for purpose. If the audit makes it clear that new tools are needed, these will be designed. The first one that may be refreshed to align with the changes in the curriculum will be e-asTTLe as it is used widely by teachers.

7. What tools will teachers use to measure progress and achievement?

There will be tools for Years 1 -3 where we know that it is important to notice and respond to foundational mathematics progress and achievement. For Years 4 upwards refreshed e-asTTLe and PATs will be used to measure critical numeracy skills and knowledge.

8. How will the Record of Learning support better maths outcomes for learners?

Developing a common approach to records of learning will support students, parents and whānau and teachers have the information they need to understand and support students' progress. The sharing of information will help the education system know what works, what needs to be improved, and where to allocate resources to better support students.

9. Will teachers get PLD or support to help them teach maths?
There will be a range of differentiated supports to meet teachers needs where they are. Curriculum Leads will support teachers to select the most appropriate PLD. Some supports are very focussed on just-in-time response, whereas others have a more long-term focus on improving teacher capability such as the DMIC programme.

10. Will there be classroom resources to help teach maths?
There will be end-to-end guidance from Years 1-13 about all aspects of mathematics and statistics teaching and learning. This will include specific guidance on maths pedagogy and pick-up-and-use classroom resources that sit alongside the refreshed learning area.

11. What support will teachers get to improve their assessment ability?
In 2021, the PLD priorities have included a focus on lifting assessment capability in the workforce. This is foundational to any focus on a particular learning area such as mathematics and will support teachers to meet the teaching standards around assessment.

12. How will the Ministry work with Initial Teacher Education providers?
The Ministry is talking with Deans of Education to ensure that the ITE Maths programme supports new teachers to meet the teaching standards, so that teachers teach in ways that ensure that all learners are making sufficient progress.

13. How will the Ministry work with the PLD workforce to support good maths teaching?
The Ministry is working with PLD providers to update the accreditation process so that it is clear which providers have mathematical expertise.

Analysis of Maths Working Group report September 2021

The focus areas requested for this report were:

- The mathematics and statistics skills and knowledge learners need to know by when
- Important cross-disciplinary links between mathematics, statistics and other discipline areas
- Considerations regarding rapid changes and growth in computer science/ICT

Acknowledgement of the need for the voice of tangata whenua when formulating advice

The writers acknowledge that they are writing from the perspective of tangata tiriti and that it is vitally importance that tangata whenua voice and cultural perspective is foundational to this work. It is also essential to ensure Tātaiako and Tapasā are embedded in all curriculum including resources and guidance.

Year by year structure

The panel suggest that three issues evident in maths and stats education in Aotearoa highlighted by national and international studies can be addressed by careful consideration of the structure of the learning area. They argued that a more closely sequenced structure will result in:

- Sufficient progress and achievement to meet our goals
- Equity in access and outcomes
- Reduction in variability of learning experiences and achievement opportunities

Teacher knowledge of mathematics and statistics for teaching is recognised as a significant factor in quality of maths teaching. Clear curriculum direction and specificity is needed drawing on evidence from developmental psychology, neuroscience, mathematics education, statistics education, mathematics and statistics disciplines.

The metaphor of a map is used to describe the maths and stats curriculum. Some advantages of setting out the curriculum expectations for each year over broader year bands. Clearly identifying learning sequence, providing requisite knowledge for concepts later in the pathway, providing opportunities to consolidate, extend and connect ideas. Teachers look for guidance in other sources when this is not provided in the curriculum. These other sources often become the "proxy curriculum".

Evidence is presented from four bodies of research:

> Early mathematics learning in primary school is very significant to future progress and has many small but important steps that could be missed if year-by-year detail is not provided.

There is a strong body of evidence that what ākonga learn early in their primary school maths journey is highly important for and predictive of their rate of progress and their achievement. Need clarity about what is important early in the pathway to maximise the potential for improved learning.

> Teacher expectations are critical to ākonga progress, and without clarity about what is possible, teachers expectations can be too low.

Evidence that teachers expect less of Māori and Pacific ākonga compared to ākonga of other ethnicities is significant when considered alongside evidence of the impact of teacher expectations on achievement in mathematics. A clear message to kaiako that ākonga are capable of more in the first 2 years of schooling than is currently indicated in the NZC is needed to accelerate progress. Large grain size of AO's in current NZC is problematic as it doesn't give enough guidance to Kaiako on what/how to plan and teach to build on prior knowledge as kaiako cannot be sure what has been covered in previous years. Lots of time spent assessing and re-teaching. The progression of concepts within and between strands need to be explicit to realise the potential for building deeper conceptual understanding.

> Teacher knowledge of mathematics and statistics for teaching is uneven, and because that knowledge is fundamental to improving learner outcomes, the curriculum needs to support and build teacher knowledge.

More scaffolding is needed to build teachers' pedagogical content knowledge and knowledge of maths and stats for teaching. Evidence suggests this will support teachers to structure programmes in connected ways and will support formative assessment.

> The nature of mathematics and statistics as disciplines demands more detail than other curriculum areas.

The structure of content (including processes) for mathematics and statistics is different to other subjects. The structure is both sequenced and interconnected. "The repertoire of constrained content in maths and stats is substantial and typically needs to be sequenced across the years of the primary school."

The mathematics skills and knowledge learners need to know by when: Years 0-13

The writers used a guiding principle of each year level having some content that was identifiably new for ākonga and extension opportunities **within the year** rather than moving to the next year. Focus on ākonga making connections to powerful ideas and understanding of mathematical processes, key competencies and cultural competencies as signposts of ākonga progress along the pathway.

"The mathematical processes are pivotal to ākonga learning and signposts inclusive of processes and competencies are needed because the processes drive learning."

The writers provide a reframing or organisation of the content strands with a rationale for this approach:

- Combining number with measurement
- Algebra as a separate strand
- Geometry as a separate strand
- Statistics remaining as a separate strand.

An overview of the important mathematics and statistics ideas developed through Te Whāriki and supporting documents for the early years is provided with a focus on connecting Te Whāriki and the NZC.

An overview of the important learning in Years 0-6 precedes the Year by Year templates, an important point to note is the Year 0 template to explicitly provide guidance on the first 2 terms of school. Overviews are provided at the beginning of the Year 7-10 and again for Years 11-13.

A statement of the overarching theme or motto for each year is given and this is then provided for each content strand. Placeholder statements are provided for these processes:

- Problems and models
- Reason and critique
- Pattern and structure.

These will need to be developed through the curriculum refresh

Signposts (requisite learning whose absence would hinder progress) are indicated in bold. The signpost will need to be further refined to inform the refresh.

Year by year templates provide a comprehensive starting point for the refresh, particularly the primary years. The upper secondary part of the pathway is the area that requires additional focus as the writing group did not include specialisation beyond mathematics education for schooling, so the link to tertiary maths rich subjects is needed.

Pedagogy – Important cross-discipline links

The working group provided advice on cross disciplinary links between mathematics and statistic and other disciplines in three different ways; using other disciplines to support learning of maths and stats, using maths and stats to support learning in other disciplines, and using a cross disciplinary integrated approach. The overall view was that the main focus of mathematics and statistics teaching is for ākonga to know, understand and do mathematics and statistics. Connecting meaningfully to other disciplines adds context and richness to the learning, however there are substantial caveats to making these links. Care is needed to ensure the integrity of both the mathematics and statistics and the other discipline(s). This is particularly challenging at secondary when the requisite subject knowledge of often diverse disciplines would be challenging for the vast majority of teachers. Examples of rich opportunities for cross discipline teaching and learning are provided, including an exemplar of cross disciplinary links between maths and stat and te reo Māori is provided as appendix 2.

Pedagogy – considering technology throughout all year levels and strands and oin relation to rapid changes and growth in computer science

The working group take a wide view of technology as "invented tool used to support the learning and doing of mathematics and statistics." They discuss the important role of technology in supporting ākonga develop mathematical and statistical ways of thinking and working. They highlight the importance of equity of access and the need to make explicit the benefits of using technology, both physical and digital.

They suggest that highlighting the computational thinking already implicit in many aspects of mathematics and statistics as a way of making computational thinking explicit within the refreshed learning area.

Appendix One

An overview of the key ideas for each year across the different templates, providing a way of viewing the progression of the main kaupapa for each strand.

Appendix Two

Gives an exemplar showing possible important cross-disciplinary links for mathematics and statistics with te reo Māori.

Appendix Three: Dr Pip Arnold's notes

Pip's response to the statistics progression (Years 0-10) that was shared for her review. Pip's response is based on the extensive research she has carried out on statistical education using the statistical enquiry cycle.

Appendix Four: Anna Fergusson's feedback

Anna's feedback is based on the statistics progression (Years 11-13) that was shared for her review. Anna is advocating for a thorough review and future focussed approach to statistics in the upper secondary part of the pathway with clear links to the study of data science at tertiary level.

Next steps for the Ministry

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