Mathematics
Pāngarau

Introduction

He kupu whakataki

The exemplars in this book should be considered in conjunction with the discussion in Book 16. A definition of mathematics and statistics in *The New Zealand Curriculum* includes the statement:

> Mathematics is the exploration and use of patterns and relationships in quantities, space, and time. Statistics is the exploration and use of patterns and relationships in data. These two disciplines are related but different ways of thinking and of solving problems. Both equip students with effective means for investigating, interpreting, explaining, and making sense of the world in which they live.[1]

The National Numeracy Strategy uses this definition:

> [T]o be numerate is to have the ability and inclination to use mathematics effectively in our lives – at home, at work, and in the community.[2]

The exemplars in this book record children participating in mathematical practices – exploring relationships and using patterns in quantities, space, and time – for a range of purposes.

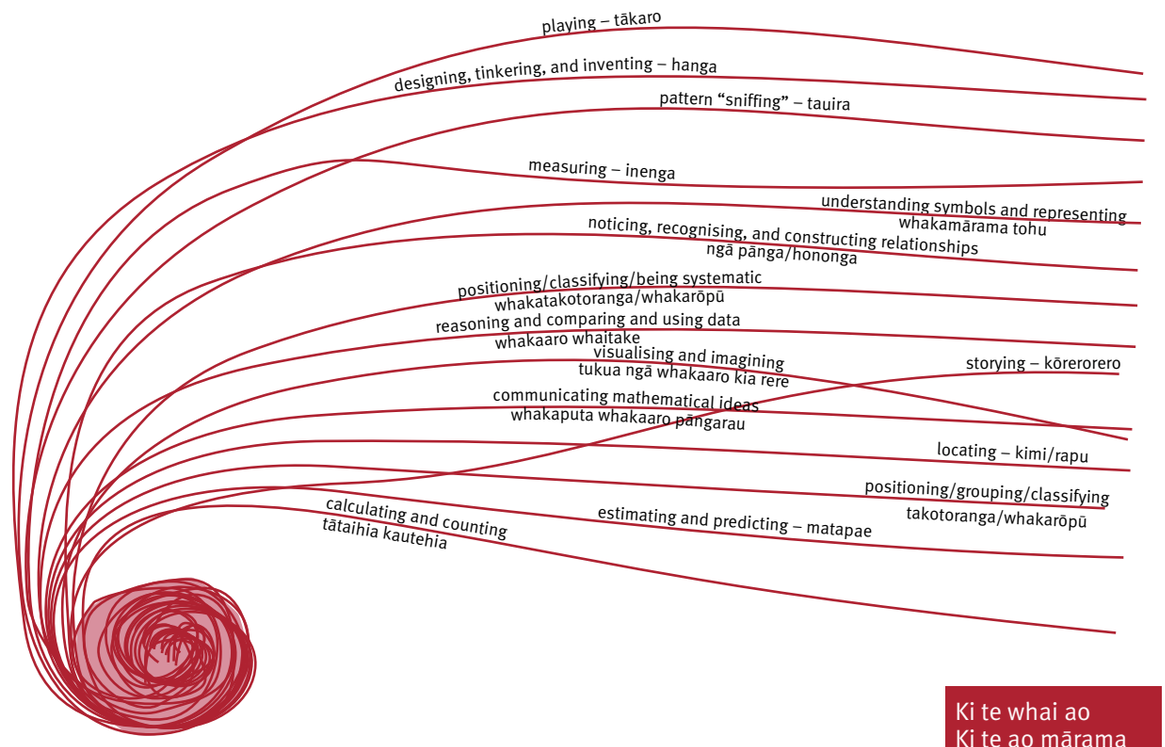
James Greeno has called this "situated knowing in a conceptual domain", and he used the workshop or the kitchen as a metaphor (see Book 16). Alan Bishop, a leading writer and researcher in mathematics education, has emphasised a cultural perspective on mathematics education that is consistent with the approach to education taken in *Te Whāriki*. He sets out six activities: counting, measuring, locating, designing, playing, and explaining.[3] He adds:

> All these activities are motivated by, and in their turn help to motivate, some environmental need. All of them stimulate, and are stimulated by, various cognitive processes, and I shall argue that all of them are significant, both separately and in interaction, for the development of mathematical ideas in any culture. Moreover all of them involve special kinds of language and representation. They all help to develop the *symbolic technology* which we call "mathematics".[4]

Discussing the importance of play to cultural life, Bishop comments, quoting Vygotsky, that "the influence of play on a child's development is enormous"[5] because it provides opportunities for abstract thinking. Barbara Rogoff also suggests that children supporting each other and learning together, a key feature of play, makes a powerful contribution to mathematical learning.[6] Bishop emphasises the playing of games. He notes that playing is "indeed a most serious business"[7] as well as a significant adult activity. Games model reality, and "it is not too difficult to imagine how the rule-governed criteria of mathematics have developed from the pleasures and satisfactions of rule-governed behaviour in games".[8] Bishop also elaborates on the activity he calls "explaining", the purpose of which is to expose relationships between phenomena. He emphasises the explanatory relationships of meaning making: finding similarity, and connections and classifications, to explain events or experiences. He notes that the diversity of languages brings culturally diverse explanatory classifications and ways of explaining.

Similarly, within the context of Aotearoa New Zealand, an early childhood mathematics working group set up in 2003 by the Ministry of Education developed "te kākano", a metaphor for describing the range of purposeful activities for developing mathematical tools and symbol systems in a bicultural environment.[9] The metaphor represents the child as te kākano (the seed), embedded in a context. The range of mathematical purposes and tools that develop is influenced by the "fertiliser" or "soil" that surrounds te kākano. These influences include teacher pedagogy, teacher content knowledge, family/whānau knowledge, and resources, all of which interact with the child's interests to privilege particular mathematical domains. The metaphor highlights the value of identifying the range of cultural purposes for mathematics within a setting.

Te kākano: Purposeful activities for the development and understanding of mathematics symbol systems and technologies



The strands in the diagram cross and interweave in different activities. For example, in one exemplar, calculating and counting, measuring, and designing might all overlap. In another, estimating and predicting might overlap with "pattern sniffing". Therefore, the names on the seed strands indicate the sorts of strategies and dispositions a teacher might notice. Each of these strands includes possibilities for increasing mathematical complexity.

A lens can be placed at any point in the diagram to look in more depth at what is happening for a particular child or group of children. Within the lens, we can see the authentic context in which an activity takes place and the specific detail of the strategies, the dispositions happening there, and the mathematical complexity involved.

Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration [BES] includes a chapter on mathematics in the early years, which is consistent with the approach taken here. It draws attention to the value of play and of everyday activities as meaningful contexts for mathematics learning, and it highlights aspects of the factors that nurture te kākano (teacher content and pedagogical knowledge, appropriate resources, and family/whānau mathematics).[10]

The mathematics exemplars in this book are viewed through one or more of the three lenses outlined in Book 16:

- a lens that focuses on assessment practices, referring to the definition of assessment as "noticing, recognising, and responding", from Book 1 of *Kei Tua o te Pae*;
- a *Te Whāriki* lens;
- a lens that focuses on the symbol systems and tools described as "mathematics".

A lens focused on assessment practices

He āta titiro ki ngā mahi aromatawai

Assessment that notices, recognises, and responds to mathematics learning in the wider sense will ensure that the mathematics in measuring, locating in space and time, designing (form, shape, and pattern), playing, and explaining are also on the curriculum agenda. Frequently the mathematics in an exemplar was not part of the teacher's analysis of the learning and has been added to the annotation for this exemplar book. The "mathematics" may not always be the focus in an analysis of the learning; sometimes other aspects of *Te Whāriki* may be recognised as important at a particular time in a child's educational journey.

However, highlighting the mathematics in documentation is an aspect of effective mathematics teaching. It encourages teachers and children and families revisiting children's portfolios to recognise and develop children's mathematical competence and continuity. Assessments can illustrate the view in *Te Whāriki* that mathematics is about symbol systems and tools for making and representing meaning and for solving and posing problems. *Te Whāriki* includes the learning outcome that "Children develop the expectation that numbers can amuse, delight, illuminate, inform, and excite".[11] Play is one way in which children will realise this expectation, and teachers will contribute to this expectation in a range of ways. A key finding from a New Zealand research project on mathematics teaching and learning in early childhood settings indicated that pedagogical documentation enhances the teaching and learning of mathematics in early childhood.[12]

A lens based on *Te Whāriki* – He tirohanga mai i *Te Whāriki*

Mathematics is woven throughout the strands in *Te Whāriki*. It is found specifically in the Communication/Mana Reo and the Exploration/Mana Aotūroa strands. The latter strand includes mathematical processes such as "setting and solving problems, looking for patterns, classifying things for a purpose, guessing, using trial and error, thinking logically and making comparisons".[13] This strand also includes spatial understandings.[14] The Communication/Mana Reo strand includes "familiarity with numbers and their uses" and "skill in using the counting system and mathematical symbols and concepts, such as numbers, length, weight, volume, shape, and pattern".[15] This strand emphasises mathematics in referring to "activities that have meaning and purpose for children" and in the phrase "for meaningful and increasingly complex purposes".[16] As for the other domains of symbol systems and tools for making meaning and communicating, the principles in *Te Whāriki* mean that family "voices" will be sought and that "funds of knowledge"[17] from home and community will be acknowledged and included in the children's portfolios.

The *Te Whāriki* perspective is that children will participate in the symbol systems and tools of mathematics for personal, social, and cultural purposes: for becoming confident and competent in culturally valued enterprises, expressing emotion, making connections across place and time, contributing their own abilities and viewpoints to the community, communicating with others (including appreciating the ways in which the available cultures communicate and represent), and making sense of their worlds.

At the same time, the possible pathways for learning that derive from the four principles of *Te Whāriki* (see Book 10) can help teachers to identify dimensions of strength as children become more interested in and involved with mathematics. Learning episodes associated with mathematical practices take on

dimensions of strength as these episodes become:

- more strongly integrated into recognised patterns, regular events, and social practices over time. The exemplar "Jack explores space" includes a number of stories about Jack's exploration of space and of his place in it. There are many everyday opportunities for him to explore his body in space (in boxes, in tunnels, and up and down steps) and to explore things in space (posting, stacking, rolling, and hiding). These opportunities provide increasing levels of challenge.
- distributed or stretched across a widening network of helpful people and enabling resources. In "Ezra explores height, balance, measurement, and number", Ezra is exploring ways in which he can be taller by trying different units for measuring his height and trying a range of ways in which he can change his height.
- connected to a greater diversity of purposes, places, and social communities.
- more mindful (as children begin to take responsibility and make up their own minds).

These last two dimensions of strength are illustrated in "Playing with repeated patterns". Jessica begins her interest during a visit to the Māori Gallery at the Auckland Museum (diversity and place). There she observes the patterns, including kōwhaiwhai, and draws them. She later constructs a complex pattern with its own personal purpose and meaning and its own unique rules and relationships (mindfulness).

A lens focused on mathematics

He āta titiro ki te pāngarau

The following are some aspects of participation, in the domain of mathematical symbols, tools, and practices, that might be noticed, recognised, responded to, recorded, and revisited. Not all these aspects are represented in the exemplars, but teachers may be able to identify them in their own local settings and write their own exemplars. In particular, when episodes are documented and revisited, children will be able to recognise their own mathematical competence.

A repertoire of mathematical practices

An indicative repertoire of practices is set out here, using the four practices outlined in Book 16 as a framework. These four practices also intersect and interconnect.

Observing and listening in to mathematical symbols, tools, and practices

Observing and listening in to mathematical symbols, tools, and practices includes watching and listening in to adults and children engaged in a range of mathematical activities. It also includes noticing cultural and local conventions to do with ways of classifying and describing patterns and relationships, using ideas like number, shape, space, time, and distance. In the exemplar "Preparing a budget and playing with numbers", a group of children and their teacher are using mathematics for a purpose: to select from a catalogue and to budget for an equipment grant. Achieving this purpose calls for using some mathematical tools, including symbols (numbers) and a calculator. Lute observes and listens in to this purposeful activity, and she later plays with the calculator, writing the numbers that appear.

Playing with mathematical symbols, tools, and practices

Playing with mathematical symbols, tools, and practices includes playing with and noticing numbers, shapes and sizes, and quantities of things. It includes trying out tools for exploring number, shape, space, time, and distance and finding out what these tools can do.

In the exemplar "Quin and quarters", Quin has been playing with symmetry and quarters, painting a pattern that she "appeared to be really happy with".

Using mathematical symbols, tools, and practices for a purpose

The "te kākano" diagram lists a number of purposeful activities for developing and understanding mathematical symbol systems and tools. This diagram has proved useful for exploring the mathematics programme in early childhood settings. Using mathematical symbols, tools, and practices for a purpose includes:

- setting and solving problems that use mathematical symbols and systems (as in the exemplar "Measuring the play dough", where Tom uses a ruler and centimetres to compare the lengths of dough and to find how far his dough can stretch);
- looking for and constructing patterns that have a "rule" or relationship that establishes the pattern (for example, symmetry; or as in the exemplar "Playing with repeated patterns", where in Jessica's repeated and sequential pattern, the legs become longer as the figures become smaller);
- connecting with a range of ways in which family and whānau do mathematics and classifying things for a purpose (as in the exemplar "Ezra explores height, balance, measurement, and number", which includes contributions about mathematical practices in Ezra's family);

- guessing, using trial and error, thinking logically, and making comparisons (as in the exemplar "Jake's survey", where Jake checks an earlier survey and systematically records the colours of bags hanging up over the lockers);
- noticing, recognising, and understanding cultural patterns (as in the exemplar "Playing with repeated patterns", where Jessica explores the koru pattern);
- noticing and recognising the purpose of significant cultural designs (as in the exemplar "Collaborative building with unit blocks", where the children explore aspects of geometric shapes);
- using mathematical systems for making meaning (as in the exemplar "Ordering by size", where Nick uses numbers and ordered sizes to make meaning and to tell a story).

Critically questioning or redesigning

Critically questioning or redesigning in mathematics includes critiquing the options for classifying and representing data for making meaning. It includes using mathematical symbols creatively and reflectively and representing the world of numbers, shapes, time, and space in personal and unique ways. In the exemplar "Jake's survey", the teachers have demonstrated a number of ways to display data from surveys. Jake has reflected on these and makes up his own mind by choosing and adapting one of the methods.